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# Saint-Petersburg olympiad in topology, 1-31 October.

Actual version of problems is situated on
`http://mathcenter.spb.ru/nikaan/olympiad/problemseng.pdf`
also in Portuguese `http://conteudo.icmc.usp.br/pessoas/sasha_a/top_olimp.pdf`

## Problems

1. Find two non-homeomorphic compact subsets $X_1$, $X_2$ of the plane, such that $X_1 \times I$ is homeomorphic to $X_2 \times I$, where $I = [0, 1]$ is a closed interval of the real line.

2. A set of four points is endowed with the minimal (in the number of open sets) topology, in which two points are open and the other two are closed. Compute the fundamental group of this space and construct its universal cover.

3. a) In the space of all triangles in the plane, is the subspace of all right triangles a deformation retract? b) Construct a deformation retract of the space of all triangles in the plane onto the subspace of all equilateral triangles.

4. Let $X$ be a connected manifold and $f : X \to S^2$ a continuous map such that $f^{-1}(x)$ is homeomorphic to $S^1$ for all the points $x$ on the sphere $S^2$. What can $H_1(X, Z)$ and $H_2(X, Z)$ be?

5. Consider $S^4 = \{x \in \mathbb{R}^5 | 1 = |x|\}$. Is it possible to choose for every point $x$ in $S^4$ a two-dimensional affine plane $P_x$, tangent to $S^4$, such that $P_x$ would continuously depend on $x$?

6. Can a Hausdorff space with a countable number of points be connected?

7. Does there exist a surjective continuous map $f : \mathbb{R}^2 \to \mathbb{R}^2$ which isn't a homeomorphism, but such that every point $x$ in $\mathbb{R}^2$ has a neighborhood $U$ for which $f : U \to f(U)$ is a homeomorphism?

8. a) Consider a square $[0, n]^2$ in the plane, for natural $n$. Erase all the points which have both coordinates non-integer. We are left with one-dimensional cell complex which we will call $X$. Find the maximal $k = k(n)$ such that for any continuous map of $X$ to $\mathbb{R}^1$ there is a point with at least $k$ preimages. b) The same for maps to $\mathbb{R}^2$ of the two-dimensional complex obtained from $[0, n]^3 \subset \mathbb{R}^3$ by erasing all the points with all coordinates non-integer.

9. Does there exist an immersion of the sphere $f : S^2 \to \mathbb{R}^3$ for which there isn't an immersion of the three-dimensional disk $g : D^3 \to \mathbb{R}^3$ such that $f = g|_{\partial D^3}$?

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## Rules

It is an olympiad for everybody. But we will grade only solutions in Russian. So if you want that somebody grades your solutions, find this person yourself.

If you still have any questions, write to nikaanspb on gmail on com.