

Lista de exercícios de SMA-333 - Cálculo III - Prof. Valdir Menegatto #13

1. Use uma série de Fourier de senos para $f(x) = x$, $x \in (0, \pi)$, para verificar que

$$x \sim \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \operatorname{sen} n\pi x, \quad 0 < x < 1.$$

2. Deduza a expansão

$$x(1+x) \sim \frac{1}{3} + \frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^n \left(\frac{2}{n^2\pi} \cos n\pi x - \frac{1}{n} \operatorname{sen} n\pi x \right), \quad 0 < x < 1.$$

3. Idem para

$$x^2 \sim 2c^2 \sum_{n=1}^{\infty} \left[\frac{(-1)^{n+1}}{n\pi} - 2 \frac{1 - (-1)^n}{(n\pi)^3} \right] \operatorname{sen} \frac{n\pi x}{c}, \quad 0 < x < c.$$

4. Idem para

$$x(1-x^2) \sim \frac{12}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} \operatorname{sen} n\pi x, \quad 0 < x < 1.$$

5. Idem para

$$x(x-1)(x-2) \sim \frac{12}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^3} \operatorname{sen} n\pi x, \quad 0 < x < 1.$$

6. Idem para

$$x(2x-x) \sim \frac{32c^2}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \operatorname{sen} \frac{(2n-1)\pi x}{2c}, \quad 0 < x < 2c.$$

7. Idem para

$$e^x \sim \frac{\operatorname{senh} c}{c} + 2 \operatorname{senh} c \sum_{n=1}^{\infty} \frac{(-1)^n}{c^2 + (n\pi)^2} \left(c \cos \frac{n\pi x}{c} - n\pi \operatorname{sen} \frac{n\pi x}{c} \right), \quad -c < x < c.$$

8. Idem para

$$\frac{c}{4} - x \sim \frac{2c}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos \frac{(4n-2)\pi x}{c}, \quad 0 < x < \frac{\pi}{2}.$$

9. Idem para

$$\cos \pi x \sim \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{n}{4n^2 - 1} \operatorname{sen} 2n\pi x, \quad 0 < x < 1.$$

Dica: Todos os exercícios desta lista podem ser resolvidos mais facilmente com a ajuda dos exercícios das listas anteriores.