$$f(x,y) = xy + e^x. \qquad F(u,v) = f(u+v,\, u-v).$$

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "g"] \arrow[rr, bend right, "F"'] & \mathbb{R}^2 \arrow[r, "f"] & \mathbb{R} \end{tikzcd}$$

$F = f \circ g$

$x = g^1(u,v) \quad y = g^2(u,v)$

$g^1(u,v) = u+v$

$g^2(u,v) = u-v$

$g(u,v) = (g^1(u,v), g^2(u,v))$

$g^i : \mathbb{R}^2 \to \mathbb{R}$

$$\nabla F = F'(u,v) = f'(g(u,v)) \circ g'(u,v) = \nabla f \circ ((g^1)', (g^2)')$$

$$\nabla F = \nabla f \circ (\nabla g^1, \nabla g^2)$$

$$\begin{pmatrix} F_u & F_v \end{pmatrix}_{1\times 2} = \begin{pmatrix} f_x & f_y \end{pmatrix}_{1\times 2} \circ \begin{pmatrix} g^1_u & g^1_v \\ g^2_u & g^2_v \end{pmatrix}_{2\times 2}$$

$$\begin{pmatrix} F_u & F_v \end{pmatrix} = \begin{pmatrix} f_x \overbrace{g^1_u}^{x_u} + f_y \overbrace{g^2_u}^{y_u} & f_x \overbrace{g^1_v}^{x_v} + f_y \overbrace{g^2_v}^{y_v} \end{pmatrix}$$

$$F(u,v) = f(u+v,\ u-v) \qquad x = g^1(u,v) = u+v$$

$$f(x,y) = xy + e^x \qquad y = g^2(u,v) = u - v$$

$$F_u(u,v) = f_x\left(g^1(u,v), g^2(u,v)\right) \cdot \overset{g^1_u}{x_u}(u,v) + f_y\left(g^1(u,v), g^2(u,v)\right) \overset{g^2_u}{y_u}(u,v)$$

$$\begin{cases} f_x(x,y) = y + e^x \\ f_y(x,y) = x \end{cases} \qquad \begin{cases} x_u = g^1_u(u,v) = 1 \\ x_v = g^1_v(u,v) = 1 \end{cases} \qquad \begin{cases} y_u = g^2_u(u,v) = 1 \\ y_v = g^2_v(u,v) = -1 \end{cases}$$

$$F_u(u,v) = \left((u-v) + e^{u+v}\right) 1 + (u+v) \cdot 1 = 2u + e^{u+v}$$

$$F_v(u,v) = f_x(u+v, u-v) \cdot x_v + f_y(u+v, u-v) \cdot y_v$$

$$= \left((u-v) + e^{u+v}\right) 1 + (u+v) \cdot (-1) = -2v + e^{u+v}$$

$$f(x,y) = (x^2+y^2+2x)^2 - (x^2+y^2) = 0$$

$f_x(x,y) = 2(x^2+y^2+2x)(2x+2) - 2x$

$f_y(x,y) = 2(x^2+y^2+2x)(2y) - 2y$

• $(0,1)$: $f(0,1) = 0$

$f_x(0,1) = 4 \neq 0 \not\Rightarrow x = h(y)$ $h: I \to J$ dif

$f_y(0,1) = 2 \neq 0 \Rightarrow y = g(x)$

$g: A \to B$ dif

$f(0,0) = 0$

$f_x(0,0) = 0$?

$f_y(0,0) = 0$

• $\underline{(-1,0)}$:

$f(-1,0) = 1 - 2 - 1 = \boxed{0}$

$f_y(-1,0) = 2(1-2)\cdot 0 - 0 = 0 \to$ pelo T.F.I não sei se a eq. $f(x,y)=0$ def. $y = g(x)$

$f_x(-1,0) = 2(1-2)(-2+2) - 2(-1) = \boxed{2 \neq 0}$

$\Rightarrow \exists!\ h: I \to J$ tq $x = h(y)$ satisfaz a eq. $f(x,y)=0$

diferenciável

$y = g(x)$ $\qquad \dfrac{dy}{dx} = -\dfrac{f_x}{f_y} = \boxed{}$

$$f(x,y) = (x^2+y^2+2x)^2 - (x^2+y^2) = 0$$

$2(x^2+y^2+2x)(2x+2yy'+2) - (2x+2yy') = 0$

$y' = \boxed{}$

$x = h(y)$ $\qquad \dfrac{dx}{dy} = -\dfrac{f_y}{f_x} = \boxed{}$

$2(x^2+y^2+2x)(2xx'+2y+2x') - 2xx'+2y = 0$

$x' = \boxed{}$

$\binom{1,\ 0}{}$