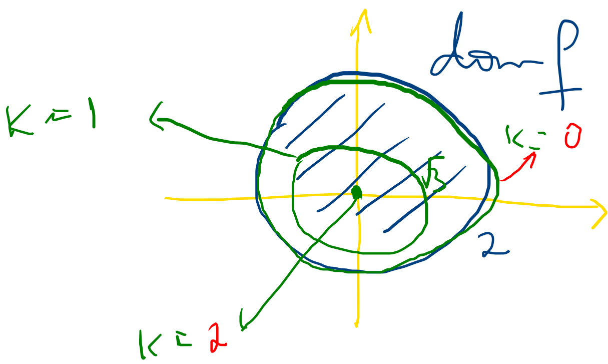
$$f(x,y) = \sqrt{4-x^2-y^2}$$

• $\mathrm{dom} f = \{(x,y) : 4-x^2-y^2 \geq 0\} = \{(x,y) : x^2+y^2 \leq 4\}$



$f(x,y) \geq 0$ $4-x^2-y^2 = 4-(x^2+y^2) \leq 4$

$f(x,y) = \sqrt{4-x^2-y^2} \leq \sqrt{4} = 2$

• $\mathrm{Im} f = [0, 2]$

• $N_k = \{(x,y) \in \mathrm{dom} f : f(x,y) = k\} \subset \mathrm{dom} f$ $k \notin \mathrm{Im} f \Rightarrow N_k = \emptyset$

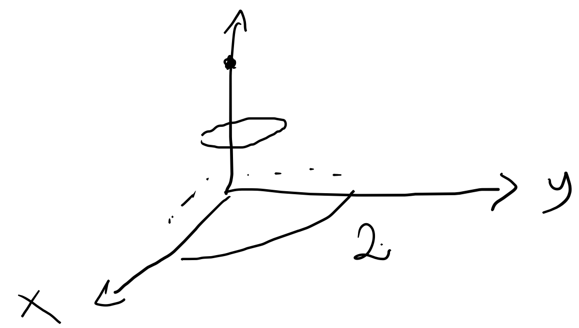
$$\Updownarrow$$

$$\sqrt{4-x^2-y^2} = k \Leftrightarrow 4-x^2-y^2 = k^2 \Leftrightarrow \boxed{x^2+y^2 = 4-k^2}$$

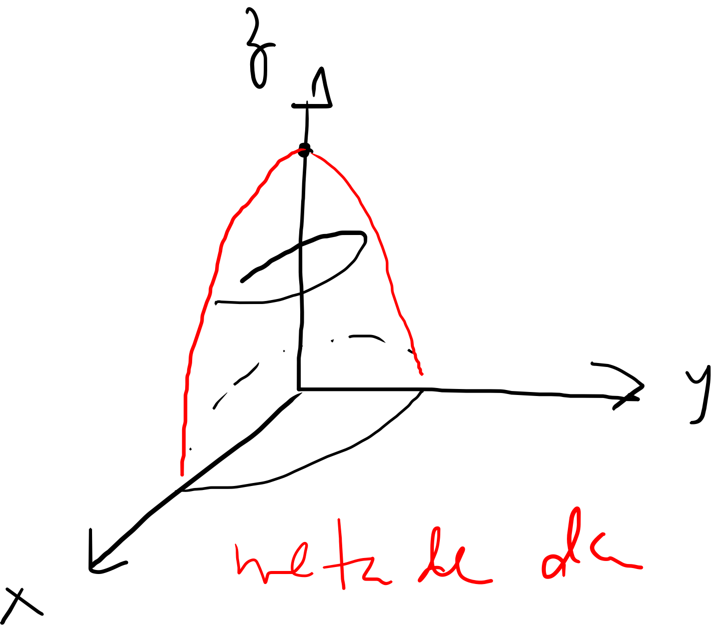
$k=0$: $x^2+y^2 = 4$

$k=1$: $x^2+y^2 = 3$

$k=2$: $x^2+y^2 = 0 \Leftrightarrow x=y=0$



$$f(x,y) = \sqrt{4-x^2-y^2}$$



metade da esfera

$$x=0 \; : \; z = \sqrt{4-y^2}$$

$$z = f(x,y) = +\sqrt{4-x^2-y^2}$$

$$z^2 = 4-x^2-y^2$$

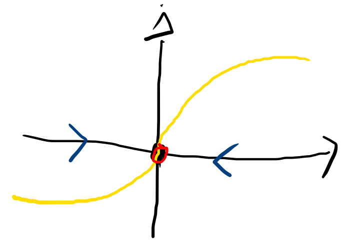
$$x^2+y^2+z^2 = 4$$

esfera de centro $(0,0,0)$ e raio 2

(a) $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$

$$\| \quad f(x,y)$$

$$\text{dom } f = \mathbb{R}^2 - \{(0,0)\}$$



$C_1 : y = 0$

$C_2 : x = y^3$

$x \neq 0$

$$\lim_{\substack{(x,y)\to(0,0)\\ C_1}} \frac{xy^3}{x^2+y^6} = \lim_{x\to 0} \frac{x\cdot 0}{x^2+0} = \lim_{x\to 0} \frac{0}{x^2} = \lim_{x\to 0} 0 = 0$$

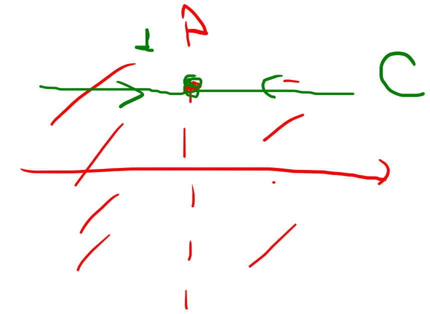
$$\lim_{\substack{(x,y)\to(0,0)\\ C_2}} \frac{xy^3}{x^2+y^6} = \lim_{y\to 0} \frac{y^3\cdot y^3}{(y^3)^2+y^6} = \lim_{y\to 0} \frac{\cancel{y^6}}{2\cancel{y^6}} = \frac{1}{2}$$

$$\times \quad \therefore \nexists \lim \frac{xy^3}{x^2+y^6}$$

(b) $\lim\limits_{(x,y)\to(0,1)} xe^{-y/x}$

$f(x,y) = x\,e^{-y/x}$

$D_f = \{(x,y) : x \neq 0\}$



$C : y = 1$

$\lim\limits_{\substack{(x,y)\to(0,1)\\ C}} x\,e^{-y/x} = \lim\limits_{x\to 0} x\,e^{-1/x} \;\nexists$

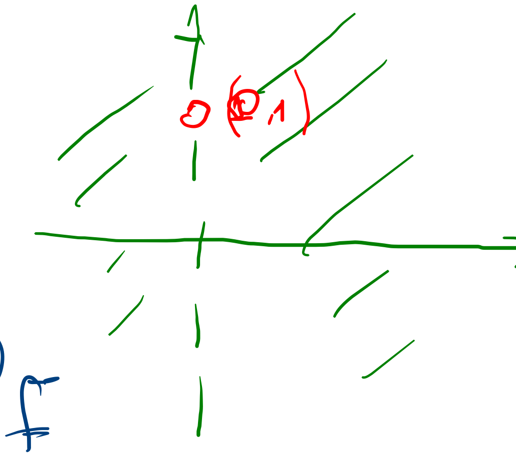
• $\lim\limits_{x\to 0^+} x\,e^{-1/x} = 0 \cdot 0 = 0$, $\quad -1/x \to -\infty$, $\quad e^{-1/x} \to 0$

• $\lim\limits_{x\to 0^-} x\,e^{-1/x} = 0\cdot\infty = \lim\limits_{x\to 0^-} \dfrac{e^{-1/x}}{1/x} \overset{L'H}{\underset{\infty/\infty}{=}} \lim\limits_{x\to 0^-} \dfrac{e^{-1/x}\cdot\frac{1}{x^2}}{-\frac{1}{x^2}} = +\infty$, $\quad x\to 0^-$, $\quad -1/x \to +\infty$, $\quad e^{-1/x} \to +\infty$

$\therefore \nexists \lim\limits_{(x,y)\to(0,1)} x\,e^{-y/x}$

(c) $\lim\limits_{(x,y)\to(0,1)} (y-1)e^{\cos(y/x)}$

$f(x,y) = (y-1)\, e^{\cos(y/x)}$ $\qquad D_f = \{(x,y) \,;\, x \neq 0\}$



• $\lim\limits_{(x,y)\to(0,1)} (y-1) = 1-1 = 0$

• $-1 \leq \cos\left(\frac{y}{x}\right) \leq 1 \, , \; \forall (x,y) \in D_f$

exponencial é crescente $\Longrightarrow \; e^{-1} \leq e^{\cos(y/x)} \leq e^{1} \, , \; \forall (x,y) \in D_f$

$\Rightarrow \; g(x,y) = e^{\cos(y/x)}$ é limitada em D_f

$\therefore \; \lim\limits_{(x,y)\to(0,1)} (y-1)\, e^{\cos(y/x)} = 0$ pelo T.A.

$f(x,y) = \sin(x^4 + 3y)$. Encontre f_{yyx}.

$f \in C^\infty(\mathbb{R}^2)$

$$f_{yyx} = f_{yxy} = f_{xyy}$$

$$f_y(x,y) = \cos(x^4 + 3y)\cdot 3$$

$$f_{yy}(x,y) = -3\,\text{sen}(x^4 + 3y)\cdot 3 = -9\,\text{sen}(x^4 + 3y)$$

$$f_{yyx}(x,y) = -9\cos(x^4 + 3y)\cdot 4x^3$$

$f \in C^k(A)$ se f é todas der. parc. até ordem k são cont. em A

$f(x,y) = x^3 + 3x^2y^7 + y^5 \in C^\infty(\mathbb{R}^2)$

$f_{xy\cancel{x}} = f_{xxy}$