1

$f : [a,b] \to \mathbb{R}$ limitada : integral de Riemann

$(a,b]$ $\quad [a,b)$ $\quad (a,b)$ $\quad (a,+\infty)$ $\quad (-\infty,b)$ $\quad (-\infty,\infty)$

f não limitada (em geral)

$f\big|_{[\delta,b]}$ Riemann integrável $\forall \delta > a$

$$\int_{-1}^{\infty} x^2\,dx = \lim_{a\to+\infty} \boxed{\int_{-1}^{a} x^2\,dx}$$

$$\int_0^2 \frac{1}{x-1}\,dx$$

$$\int x^{-\alpha}\, dx = \begin{cases} \ln x + c, & \alpha = 1 \\ \dfrac{x^{-\alpha+1}}{-\alpha+1} + c, & \alpha \neq 1 \end{cases}$$

$\alpha < 0$

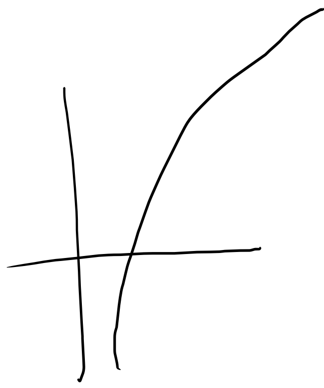
$\int_a^{\infty} x^{-\alpha}\, dx$ pol.

$\int_a^{\infty} \frac{1}{x^\alpha}\, dx$ è

- CONV. qdo $\alpha > 1$
- DIV. qdo $\alpha \leq 1$

$(a > 0)$

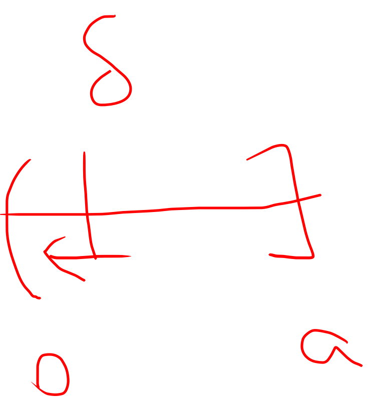
$$\int_a^{\infty} \frac{1}{x^\alpha}\, dx = \begin{cases} \lim\limits_{M\to\infty} \left[\dfrac{x^{1-\alpha}}{1-\alpha}\right]_a^M = \lim\limits_{M\to\infty} \dfrac{1}{1-\alpha}\left[\underbrace{M^{1-\alpha}}_{>0} - a^{1-\alpha}\right] = +\infty & se\ \alpha < 1 \\ \lim\limits_{M\to\infty} [\ln(x)]_a^M = \lim\limits_{M\to\infty} [\ln(M) - \ln(a)] = +\infty & se\ \alpha = 1 \\ \lim\limits_{M\to\infty} \left[\dfrac{x^{\overbrace{1-\alpha}^{<0}}}{1-\alpha}\right]_a^M = \lim\limits_{M\to\infty} \dfrac{1}{1-\alpha}\left[\dfrac{1}{\underbrace{M^{\alpha-1}}_{>0}} - a^{1-\alpha}\right] = \dfrac{a^{1-\alpha}}{\alpha-1} & se\ \alpha > 1 \end{cases}$$



$1-\alpha > 0 \iff 1 > \alpha \iff \alpha < 1$

$1-\alpha < 0 \iff \alpha > 1$

$$\int_0^a \frac{1}{x^\alpha}\,dx \ \text{è} \begin{cases} \text{CONV. se } 0 < \alpha < 1 \\ \text{DIV. se } \alpha \geq 1 \end{cases}$$



$$\int_0^a \frac{1}{x^\alpha}\,dx = \begin{cases} \lim\limits_{\delta\to 0^+} \left[\dfrac{x^{\overset{>0}{1-\alpha}}}{1-\alpha}\right]_\delta^a = \lim\limits_{\delta\to 0^+} \dfrac{1}{1-\alpha}\left[a^{1-\alpha} - \cancel{\delta^{1-\alpha}}^{\to 0}\right] = \dfrac{a^{1-\alpha}}{1-\alpha} & se\ 0<\alpha<1 \\ \lim\limits_{\delta\to 0^+} [\ln(x)]_\delta^a = \lim\limits_{\delta\to 0^+} (\ln(a) - \ln(\delta)) = +\infty & se\ \alpha = 1 \\ \lim\limits_{\delta\to 0^+} \left[\dfrac{x^{\overset{<0}{1-\alpha}}}{1-\alpha}\right]_\delta^a = \lim\limits_{\delta\to 0^+} \dfrac{1}{1-\alpha}\left[\overbrace{a^{1-\alpha}}^{\frac{1}{a^{\alpha-1}}} - \cancel{\dfrac{1}{\delta^{\alpha-1}}}^{\to +\infty}\right] = +\infty & se\ \alpha > 1 \end{cases}$$

$$\int_0^\infty \frac{1}{\sqrt{x}\,(1+x)}\,dx$$

CONVERGE

$$\underbrace{\int_0^\infty \frac{1}{x^\alpha}\,dx}_{} = \underbrace{\int_0^7 \frac{1}{x^\alpha}\,dx}_{} + \underbrace{\int_7^\infty \frac{1}{x^\alpha}\,dx}_{}$$

C	$\Leftarrow$	C	e	C
D	$\Leftarrow$	D		
D	$\Leftarrow$			D

NÃO OCORRE

(a) $\displaystyle\int_1^{\infty} \frac{sen^2(x)}{x^2+1}\,dx$, é CONV.

pois:

$0 \leq f \leq g \Rightarrow 0 \leq \int_a^{\infty} f \leq \int_a^{\infty} g$

$C \Leftarrow C$

$D \Rightarrow D$

$$0 \leq \frac{sen^2 x}{1+x^2} \leq \frac{1}{x^2}, \ \forall x \in \mathbb{R} \ \text{e} \ \int_1^{\infty} \frac{1}{x^2}\,dx \ \text{é CONV.}$$

$(\alpha = 2 > 1)$

$|sen\, x| \leq 1$

$sen^2 x \leq 1 \quad \forall x \in \mathbb{R} \Longrightarrow \dfrac{sen^2 x}{1+x^2} \leq \dfrac{1}{1+x^2} \leq \dfrac{1}{x^2}, \forall x$

$(1+x^2 > 0) \quad \dfrac{sen^2 x}{1+x^2} \leq \dfrac{1}{1+x^2}$

$1+x^2 > 0$

$1+x^2 \geq x^2, \ \forall x$

(c) $\int_{-\frac{1}{2}}^{0} \frac{1}{x^{\frac{2}{3}}}\, dx.$

$$\int_{-1/2}^{0} \frac{1}{\sqrt[3]{x^2}}\, dx = ☆$$

$f(x) = \frac{1}{\sqrt[3]{x^2}}$

$\text{dom} f = \mathbb{R} - \{0\}$

$f(x) = x^{1/3}$ $\text{dom} f = \mathbb{R}$

$f(x) = x^{3/2} = \sqrt{x^3}$

$\text{dom} f = [0, \infty)$

$u = -x$

$du = -1\, dx$

$$\begin{cases} x = -1/2 \Rightarrow u = 1/2 \\ x = 0 \Rightarrow u = 0 \end{cases}$$

$$☆ = -\int_{1/2}^{0} \frac{1}{u^{2/3}}\, du = -\left(-\int_{0}^{1/2} \frac{1}{u^{2/3}}\, du\right) = \int_{0}^{1/2} \frac{1}{u^{2/3}}\, du \text{ CONV.}$$

$\alpha = \frac{2}{3} < 1$

$$\int_{-1/2}^{0} \frac{1}{x^{3/2}}\, dx$$

NÃO FAZ SENTIDO