1. Calcule $\int \frac{x-3}{x^2+2x+4}dx.$

$$\frac{Ax+B}{x^2+2x+4}$$

$$\int \frac{x-3}{x^2+2x+4}\,dx = \int \frac{x-3}{(x+1)^2+3}\,dx =$$

$u = x+1$

$du = 1\,dx$

$$\int \frac{u-1-3}{u^2+3}\,du$$

$$= \int \frac{u-4}{u^2+3}\,du = \int \left(\frac{u}{u^2+3} - \frac{4}{u^2+3}\right)du =$$

$v = u^2+3$

$dv = 2u\,du$

$$\int \frac{u\,du}{u^2+3} = \int \frac{dv}{2v} = \frac{1}{2}\int \frac{1}{v}\,dv = \frac{1}{2}\ln|v|+c$$

$v = u^2+3$

$dv = 2u\,du$

$$\int \frac{4}{u^2+3}\,du = 4\int \frac{1}{3+u^2}\,du = \frac{4}{3}\int \frac{1}{1+\frac{u^2}{3}}\,du = \frac{4}{3}\int \frac{1}{1+\left(\frac{u}{\sqrt{3}}\right)^2}\,du$$

$$= \frac{4}{3}\sqrt{3}\int \frac{1}{1+v^2}\,dv = \frac{4\sqrt{3}}{3}\operatorname{arctg} v + c$$

$v = \frac{u}{\sqrt{3}}$

$dv = \frac{1}{\sqrt{3}}\,du$

$$(*) = \frac{1}{2}\ln(u^2+3) - \frac{4\sqrt{3}}{3}\operatorname{arctg}\left(\frac{u}{\sqrt{3}}\right) = \frac{1}{3}\ln((x+1)^2+3) - \frac{4\sqrt{3}}{3}\operatorname{arctg}\frac{x+1}{\sqrt{3}} + C$$

2. Calcule $\int \frac{x+1}{x^2+2x+3}dx$.

$$u = x^2 + 2x + 3$$

$$du = (2x+2)dx$$

$$du = 2(x+1)dx$$

$$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2+2x+3| + C$$

3. Calcule $\int \tan^3(x)\sec^4(x)dx.$ $\qquad \sec^2 x = \operatorname{tg}^2 x + 1; \qquad (\sec x)' = \sec x \operatorname{tg} x; \qquad (\operatorname{tg} x)' = \sec^2 x$

$u = \operatorname{tg} x$

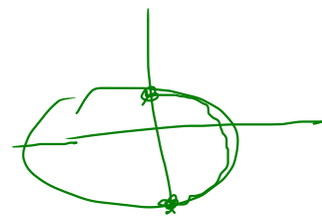
$du = \sec^2 x\, dx$

$\int \operatorname{tg}^3(x)\sec^2 x \sec^2 x\, dx = \int \operatorname{tg}^3 x (\operatorname{tg}^2 x + 1) \sec^2 x\, dx$ $\qquad \operatorname{tg}^2 x + 1$

$= \int u^3(u^2+1)\, du = \int u^5 + u^3\, du = \frac{u^6}{6} + \frac{u^4}{4} + C$

$= \frac{\operatorname{tg}^6 x}{6} + \frac{\operatorname{tg}^4 x}{4} + C$

4. Calcule $\int \frac{1}{x\sqrt{4-x^2}}dx.$ $\sqrt{a^2-x^2}$: $x = a\sin\theta$, $\theta \in [-\pi/2, \pi/2]$, $dx = a\cos\theta d\theta$ $(a>0)$



$x = 2\sin\theta \qquad dx = 2\cos\theta\, d\theta$

$$\int \frac{1}{2\sin\theta\sqrt{4-4\sin^2\theta}}\, 2\cos\theta\, d\theta = \int \frac{\cos\theta}{\sin\theta\, 2\sqrt{\cos^2\theta}}\, d\theta =$$

$$= \frac{1}{2}\int \frac{\cos\theta}{\sin\theta\,|\cos\theta|}\, d\theta = \frac{1}{2}\int \frac{1}{\sin\theta}\, d\theta = \frac{1}{2}\int \frac{\sin\theta}{\sin^2\theta}\, d\theta$$

$\cos\theta > 0$

$|\cos\theta| = \cos\theta$

$$= \frac{1}{2}\int \frac{\sin\theta\, d\theta}{1-\cos^2\theta} =$$

$u = \cos\theta$

$$\int \frac{\sin\theta}{1-\cos^2\theta}\,d\theta = \int \frac{-1}{1-u^2}\,du$$

$u = \cos\theta$

$du = -\sin\theta\,d\theta$

$$= \int \frac{-1}{(1-u)(1+u)}$$

$du = \ldots$

terminar