

4. Calculate  $\int \frac{1}{x^3 - 5x^2 + 8x - 4} dx$ .

$$x = 2 : 8 - 20 + 16 - 4 = 0$$

$$\begin{array}{r}
 x^3 - 5x^2 + 8x - 4 \\
 - x^3 + 2x^2 \\
 \hline
 -3x^2 + 8x - 4 \\
 + 3x^2 - 6x \\
 \hline
 2x - 4 \\
 - 2x + 4 \\
 \hline
 0
 \end{array}
 \quad \left| \begin{array}{l}
 x - 2 \\
 \hline
 x^2 - 3x + 2 \\
 \hline
 5 = 3 \quad x_1 = 2 \\
 p = 2 \quad x_2 = 1
 \end{array} \right.$$

$$\frac{1}{x^3 - 5x^2 + 8x - 4} = \frac{1}{(x-2)(x^2 - 3x + 2)}$$

$$= \frac{1}{(x-2)(x-2)(x-1)}$$

$$= \frac{1}{(x-1)(x-2)^2}$$

$$\frac{1}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$= \frac{A(x-2)^2 + B(x-1)(x-2) + C(x-1)}{(x-1)(x-2)^2}, \quad x \neq 1, x \neq 2$$

$$1 = A(x-2)^2 + B(x-1)(x-2) + C(x-1), \quad \forall x \Leftrightarrow$$

$$1 = (A+B)x^2 + (-4A-3B+C)x + (4A+2B-C)$$

$$\begin{cases} A+B=0 \\ -4A-3B+C=0 \\ 4A+2B-C=1 \end{cases}$$

$$\boxed{A=1, B=-1, C=1}$$

$$\mathbf{I} = \int \frac{1}{x-1} dx + \int \frac{-1}{x-2} dx + \int \frac{1}{(x-2)^2} dx$$

$$= \ln|x-1| - \ln|x-2| + \frac{(x-2)^{-1}}{-1} + C$$

$$\left\{ \begin{array}{l} u = x-2 \\ du = dx \\ \int u^{-2} du \end{array} \right.$$