1. Qual é a diferença entre: $\int_a^b f(t)dt$ (integral definida), $\int f(t)dt$ (integral indefinida) e $\int_a^x f(t)dt$?

$\int_a^b f(t)\,dt =$ um número real

$a, b =$ cte

$\int f(t)\,dt =$ família de funções

x variável

$\int_a^x f(t)\,dt =$ é uma função de x

$= F(x) - F(a) = g(x)$

$F' = f$

(a) Calcule $\frac{d}{dx}\left(\int_0^{x^3} f(t)dt\right)$

TFC - parte 2 : $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$

$g(x) = \int_0^{x^3} f(t)\,dt$

$g(x) = F \circ G(x)$

$\begin{cases} F(x) = \int_0^x f(t)dt \\ G(x) = x^3 \\ F'(x) = f(x) \\ G'(x) = 3x^2 \end{cases}$

$g'(x) = F'(G(x)) \cdot G'(x)$

$= f(x^3) \cdot 3x^2$

$= \operatorname{sen}(x^3) \cdot 3x^2$

(b) Calcule $\int_0^{2\pi} f(x)dx$.

TFC - parte 2 : $\int_a^b f(x)\,dx = F(x)\Big|_a^b$

$F' = f$

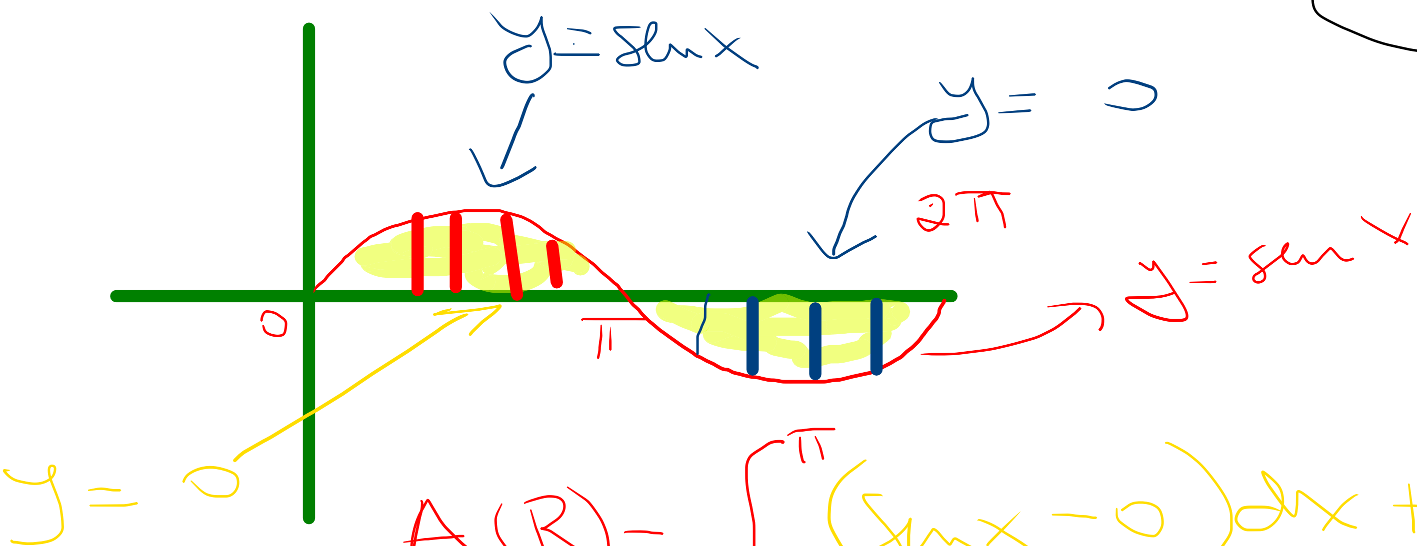
$$\int_0^{2\pi} \operatorname{sen} x \, dx = -\cos x \Big|_0^{2\pi} = -\cos 2\pi - (-\cos 0)$$

$$(\cos x)' = -\operatorname{sen} x \qquad = -1 + 1 = \boxed{0}$$

(c) Calcule a área entre o gráfico de f e o eixo-x para $x \in [0, 2\pi]$.

$$\boxed{\int_0^{2\pi} \text{sen}(x)\,dx = 0}$$

$$A(R) = \int_a^b |f(x) - g(x)|\,dx$$



$$A(R) = \int_0^{\pi} (\text{sen}\,x - 0)\,dx + \int_{\pi}^{2\pi} (0 - \text{sen}\,x)\,dx$$

$$= -\cos x \Big|_0^{\pi} + (\cos x)\Big|_{\pi}^{2\pi} =$$

$$= -\cos\pi + \cos 0 + \cos 2\pi - \cos\pi = 1 + 1 + 1 + 1 = \boxed{4}\ u^2$$

3. Determine o valor médio $M(f)$ da função $f(x) = \sqrt{4-x^2}$ em $[-2,2]$.

$$M(f) = \frac{\int_a^b f(x)\,dx}{b-a}$$

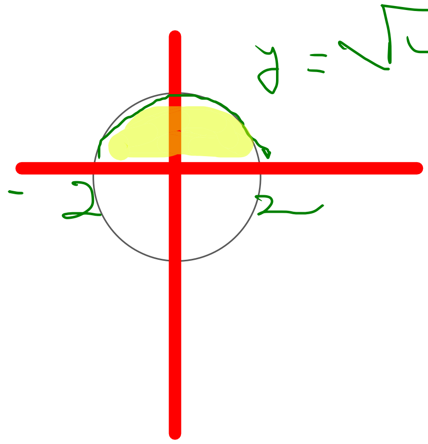
valor médio de f em $[a,b]$

$$M(f) = \frac{\int_{-2}^{2}\sqrt{4-x^2}\,dx}{2-(-2)} = \frac{1}{4}\int_{-2}^{2}\sqrt{4-x^2}\,dx$$

$y = f(x) = +\sqrt{4-x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$



$y = \sqrt{4-x^2}$

$$= \frac{1}{4}\,\frac{1}{2}\,\pi(2)^2 = \frac{\pi}{2}$$

(a) $\int \ln(t)dt$

$$\boxed{\int \mathbf{u}\,\mathbf{dv} = \mathbf{uv} - \int \mathbf{v}\,\mathbf{du}}$$

$u = \ln t \Rightarrow du = \frac{1}{t}dt$

$dv = dt \Rightarrow v = \int 1\,dt = t$

$\int u\,dv = \int \ln t\,dt = t\ln t - \int t\,\frac{1}{t}dt = t\ln t - \int 1\,dt$

$= t\ln t - t + C$

(b) $\int e^x \sin(x)dx$

$\boxed{\int \mathbf{u\,dv = uv - \int v\,du}}$

$u = e^x \Longrightarrow du = e^x dx$

$dv = \operatorname{sen} x\, dx \Rightarrow v = \int \operatorname{sen} x\, dx = -\cos x$

$\int u\,dv = \int e^x \operatorname{sen} x\, dx = -e^x \cos x + \int e^x \cos x\, dx$ $*1$

$\int e^x \cos x\, dx = *$

$u = e^x \Longrightarrow du = e^x dx$

$dv = \cos x\, d\cancel{x} \Rightarrow v = \int \cos x\, dx = \operatorname{sen} x$

$* = e^x \operatorname{sen} x - \int \operatorname{sen} x\, e^x dx$ $*2$

$\int e^x \operatorname{sen} x\, dx \overset{*1}{=} -e^x \cos x + e^x \operatorname{sen} x -$
$*2$
$\int e^x \operatorname{sen} x\, dx \Rightarrow$

$\Rightarrow 2\int e^x \operatorname{sen} x\, dx = -e^x \cos x + e^x \operatorname{sen} x$

$\Downarrow$

$\int e^x \operatorname{sen} x\, dx = \frac{1}{2}\left[e^x \operatorname{sen} x - e^x \cos x\right] + C$