

MAX
MIN
ABS

$$\left. \begin{array}{l} D \text{ compacto} \\ f \text{ cont.} \end{array} \right\}$$

frontiera $D = g(x,y) = c$
 $g(x,y) = x^2 + y^2 = 4$

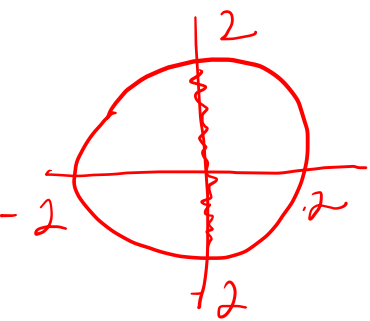
• P.C. no interior D

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases}$$



• f | fronteira

$$\left. \begin{array}{l} \text{Lagrange} \\ \text{ou} \end{array} \right\} \begin{cases} \nabla f = \lambda \nabla g \\ g = c \end{cases} \quad (P_2)$$



$$f(x,y) = x^2 + y^2 + x^2 y + 4$$

$$x = \pm \sqrt{4 - y^2}$$

fronteira :

$$x^2 + y^2 = 4 \Leftrightarrow x^2 = 4 - y^2$$

$f|_{\text{fronteira}}$

$$f(\pm\sqrt{4-y^2}, y) = \underbrace{(4-y^2)}_{x^2} + y + \underbrace{(4-y^2)}_{x^2} y + 4$$

$g(y) =$

$$g(y) = 8 + 4y - y^3, \quad y \in [-2, 2]$$

$$g'(y) = 4 - 3y^2 = 0 \Rightarrow y = \pm \frac{2}{\sqrt{3}} = \pm \frac{2\sqrt{3}}{3} \Rightarrow x = \pm \sqrt{\frac{8}{3}} = \pm \frac{2\sqrt{6}}{3}$$

$y = 2 \rightarrow x = 0$
 $y = -2 \rightarrow x = 0$

$$f(0,0) = 4$$

$$f(x,y) = x^2 + y^2 + x^2 y + 4$$

\mathbb{R}^2

MAX. ABS.
NÃO TEM

$$f(3,3) = -5$$

$$P.C : \begin{cases} f_x = 2x + 2xy = 0 \\ f_y = 2y + x^2 = 0 \end{cases}$$

$$P_1 = (0,0)$$

$$P_2 = (\sqrt{2}, -1), P_3 = (-\sqrt{2}, -1)$$

$$\det H(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 2 + 2y & 2x \\ 2x & 2 \end{vmatrix}$$

$$d(H(0,0)) = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0$$

$(0,0)$ é m. local

$$d(H(\pm\sqrt{2}, -1)) = \begin{vmatrix} 0 & -2\sqrt{2} \\ -2\sqrt{2} & 2 \end{vmatrix} = -8 < 0$$

$(\pm\sqrt{2}, -1)$ p. sela