
1 Funções hiperbólicas

Definição:

$$\begin{aligned}\mathbf{Sh}(x) &= \frac{e^x - e^{-x}}{2}, & \mathbf{D}_{\mathbf{Sh}} &= \mathbb{R}, & \mathbf{Im}_{\mathbf{Sh}} &= \mathbb{R} \\ \mathbf{Ch}(x) &= \frac{e^x + e^{-x}}{2}, & \mathbf{D}_{\mathbf{Ch}} &= \mathbb{R}, & \mathbf{Im}_{\mathbf{Ch}} &= [1, \infty) \\ \mathbf{Th}(x) &= \frac{\mathbf{Sh}(x)}{\mathbf{Ch}(x)}, & \mathbf{D}_{\mathbf{Th}} &= \mathbb{R}, & \mathbf{Im}_{\mathbf{Th}} &= (-1, 1)\end{aligned}$$

Relações:

$$\begin{aligned}\mathbf{Ch}^2(x) - \mathbf{Sh}^2(x) &= 1, \\ \mathbf{Ch}(2x) &= \mathbf{Ch}^2(x) + \mathbf{Sh}^2(x), & \mathbf{Sh}(2x) &= 2\mathbf{Sh}(x)\mathbf{Ch}(x)\end{aligned}$$

$$\begin{aligned}(\mathbf{Sh})'(x) &= \mathbf{Ch}(x) && \text{em } \mathbb{R} \\ (\mathbf{Ch})'(x) &= \mathbf{Sh}(x) && \text{em } \mathbb{R} \\ (\mathbf{Th})'(x) &= \mathbf{Sch}^2(x) && \text{em } \mathbb{R}\end{aligned}$$

Inversas:

$$\mathit{SettSh} = \mathit{Sh}^{-1}$$

$$\mathit{SettCh} = (\mathit{Ch}^*)^{-1} \quad \text{onde } \mathit{Ch}^* : [0, \infty) \rightarrow [1, \infty) : x \mapsto \mathit{Ch}(x)$$

$$\mathit{SettTh} = (\mathit{Th}^*)^{-1} \quad \text{onde } \mathit{Th}^* : \mathbb{R} \rightarrow (-1, 1) : x \mapsto \mathit{Th}(x)$$

Fórmula explícita para as inversas:

$$\text{SettSh}(x) = \ln(x + \sqrt{x^2 + 1}), \quad x \in \mathbb{R}$$

$$\text{SettCh}(x) = \ln(x + \sqrt{x^2 - 1}), \quad x \in [1, \infty)$$

$$\text{SettTh}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right), \quad x \in (-1, 1)$$

$$\text{SettCoTh}(x) = \frac{1}{2} \ln \left(\frac{1+x}{x-1} \right), \quad x \in (-\infty, -1) \cup (1, \infty)$$

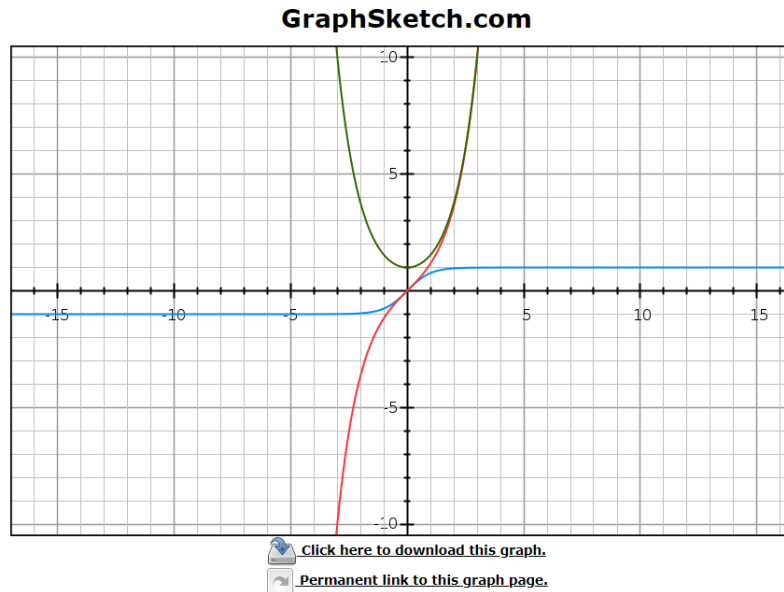
Derivadas das inversas:

$$(\text{SettSh})'(x) = \frac{1}{\sqrt{x^2 + 1}} \quad \text{em } \mathbb{R}$$

$$(\text{SettCh})'(x) = \frac{1}{\sqrt{x^2 - 1}}, \quad \text{em } (1, \infty)$$

$$(\text{SettTh})'(x) = \frac{1}{1-x^2}, \quad \text{em } (-1, 1)$$

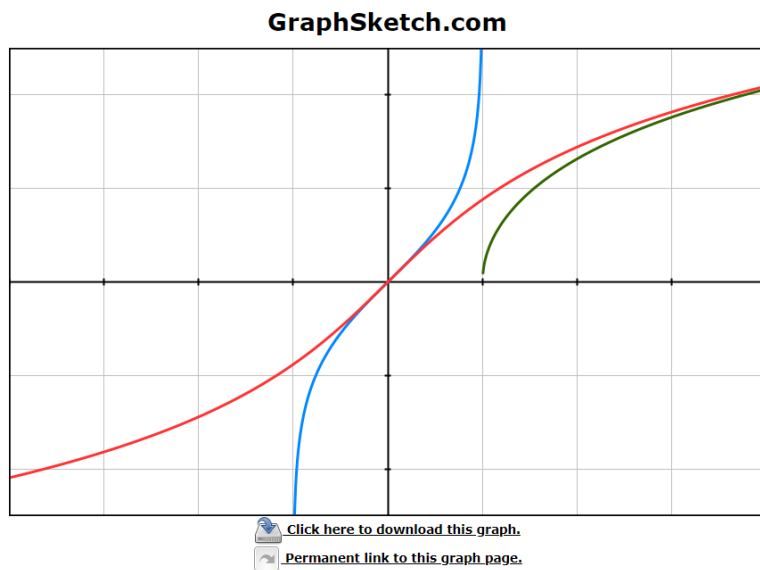
$$(\text{SettCoTh})'(x) = \frac{1}{1-x^2}, \quad \text{em } (-\infty, -1) \cup (1, \infty)$$



le: Functions Parametric
 er Graph Equations: Settings:

1. <input type="checkbox"/> $f(x) = \tanh(x)$	X Range: <input type="text" value="-17"/> to <input type="text" value="17"/>
2. <input type="checkbox"/> $f(x) = \sinh(x)$	Y Range: <input type="text" value="-10.5"/> to <input type="text" value="10.5"/>
3. <input type="checkbox"/> $f(x) = \cosh(x)$	X Tick Distance: <input type="text" value="1"/>
	Y Tick Distance: <input type="text" value="1"/>

Figura 1: Gráficos das funções hiperbólicas: Sh (vermelho), Ch (verde) e Th (azul).



le: Functions Parametric
 er Graph Equations: Settings:

1. <input type="checkbox"/> $f(x) = \operatorname{atanh}(x)$	X Range: <input type="text" value="-4"/> to <input type="text" value="4"/>
2. <input type="checkbox"/> $f(x) = \operatorname{asinh}(x)$	Y Range: <input type="text" value="-2.5"/> to <input type="text" value="2.5"/>
3. <input type="checkbox"/> $f(x) = \operatorname{acosh}(x)$	X Tick Distance: <input type="text" value="1"/>
	Y Tick Distance: <input type="text" value="1"/>

Figura 2: Gráficos das funções hiperbólicas inversas: $SettSh$ (vermelho), $SettCh$ (verde) e $SettTh$ (azul).