Redes Complexas

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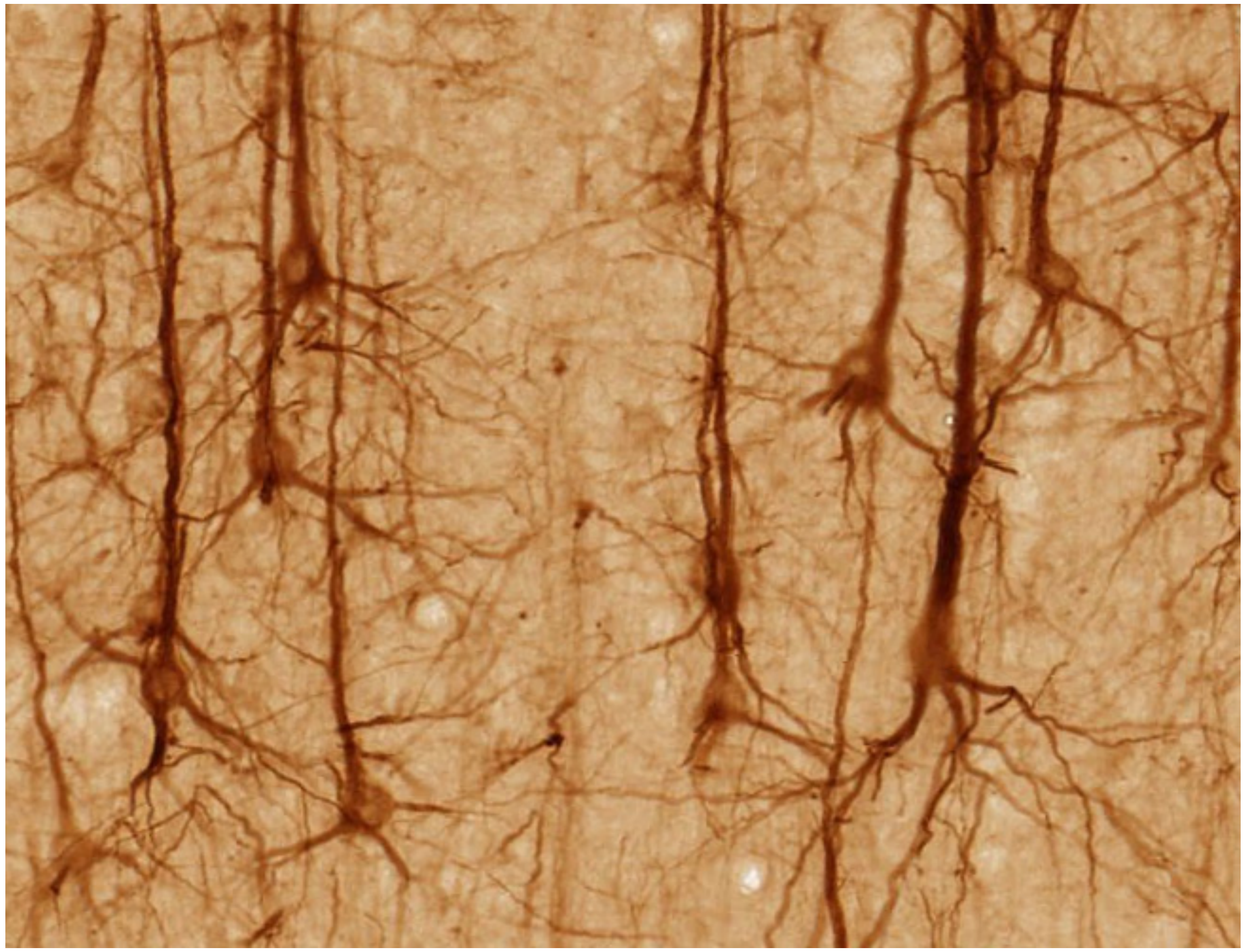
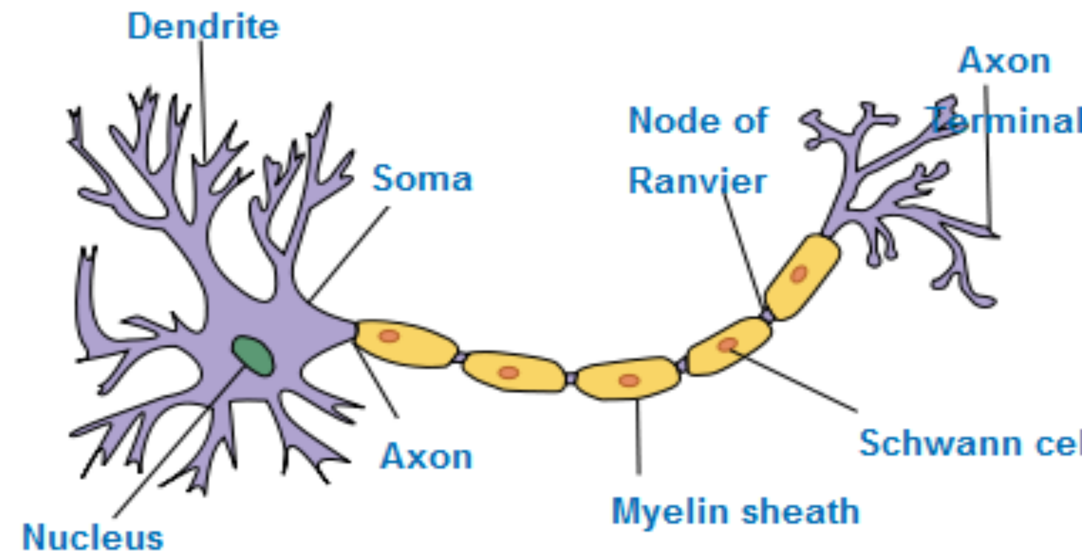
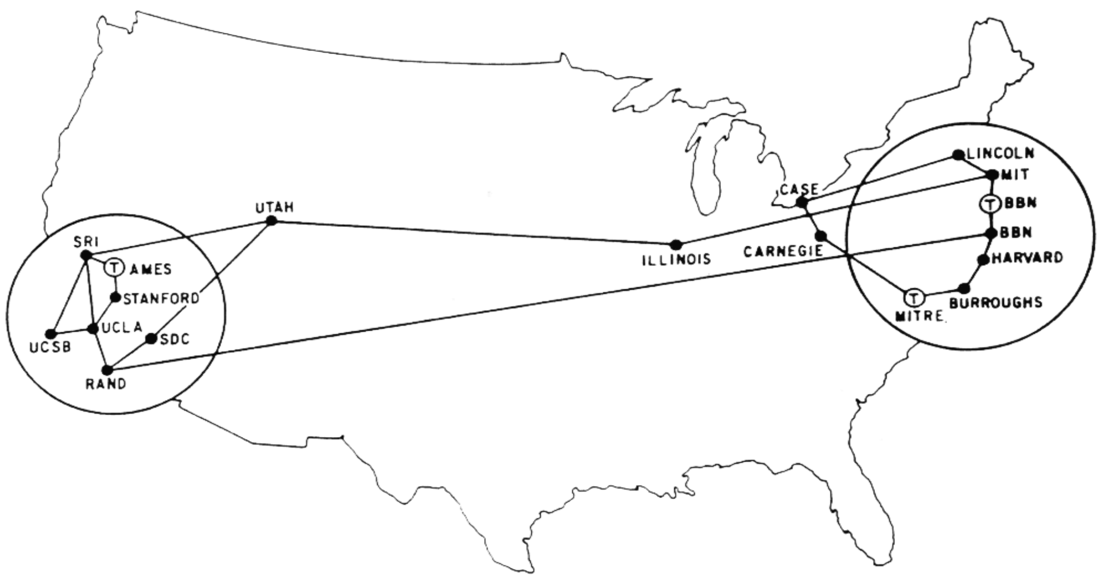
O que esses sistemas tem em comum?

Cérebro

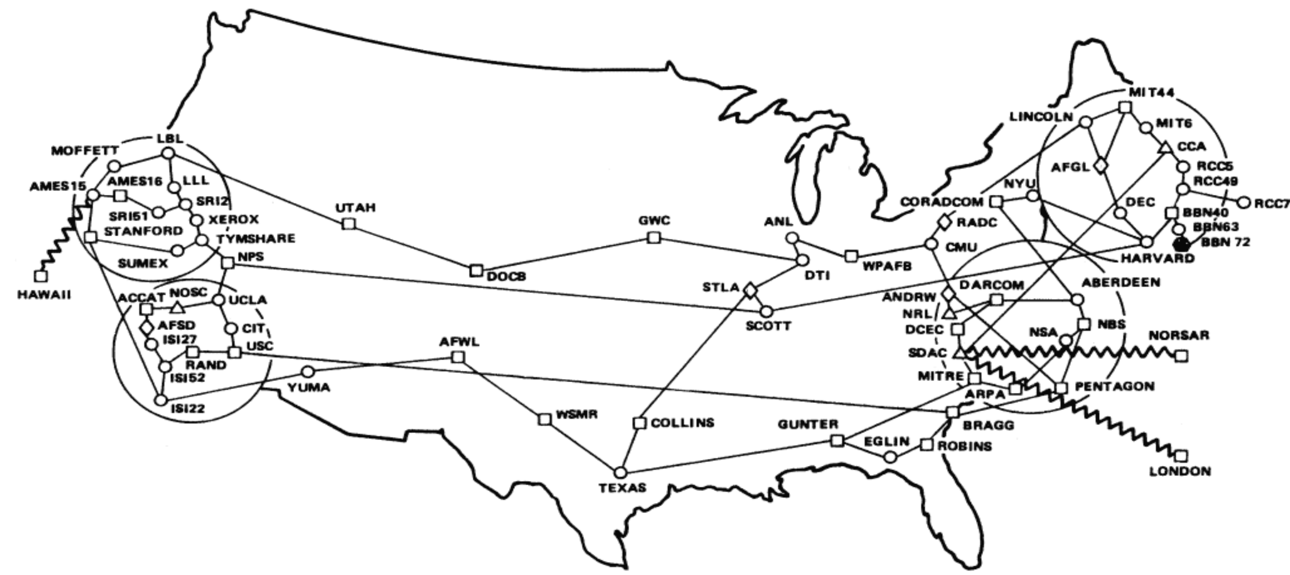



Dendrite
Axon
Terminal
Node of
Ranvier
Soma
Schwann cell
Axon
Myelin sheath
Nucleus

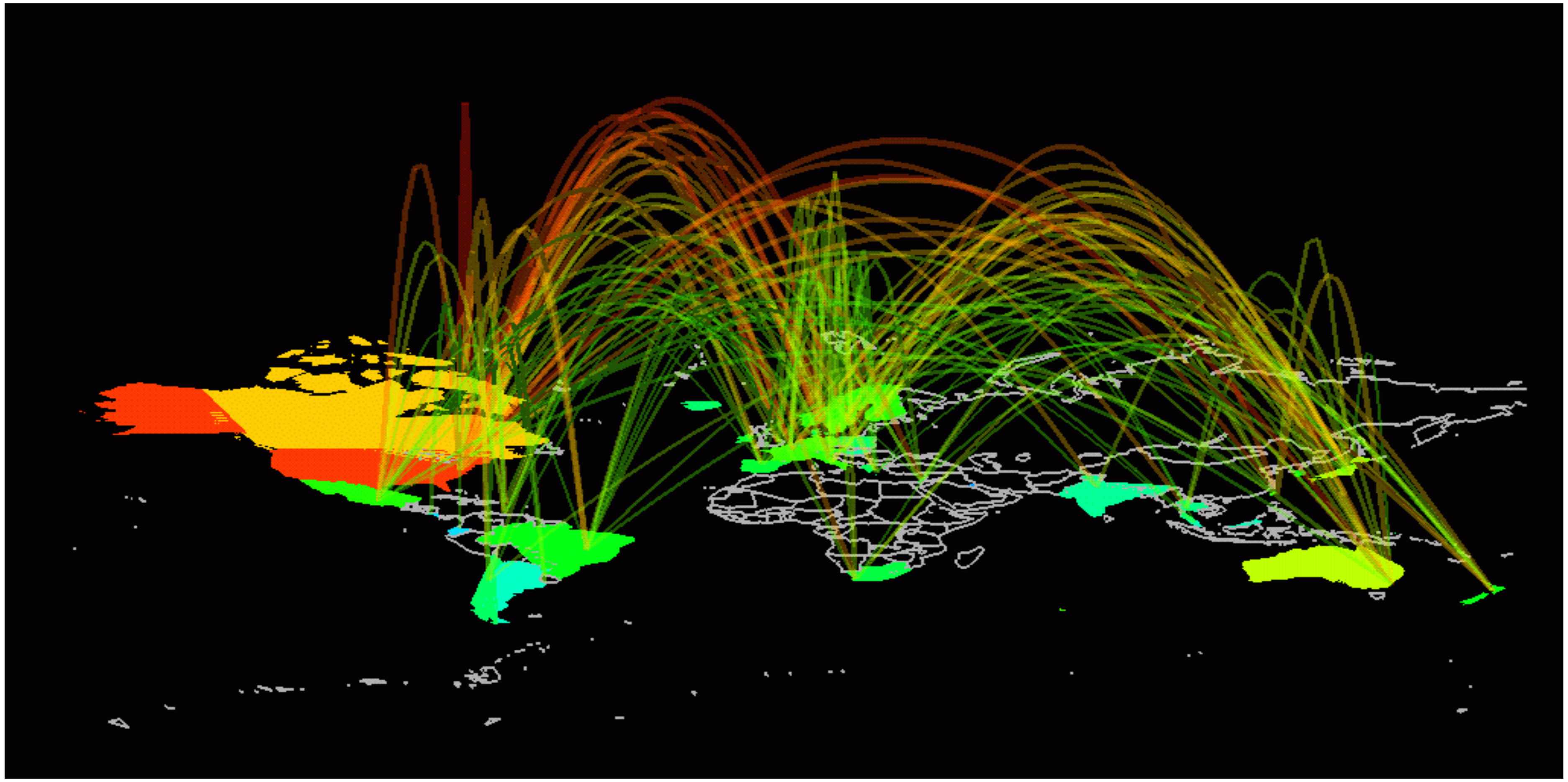
Internet



SRI
AMES
STANFORD
UCLA
UCSB
SDC
RAND
UTAH
ILLINOIS
CASE
CARNEGIE
LINCOLN
MIT
BBN
BBN
HARVARD
BURROUGHS
MITRE



MOFFETT
LBL
AMES15
AMES16
LLL
SRI2
SRI51
XEROX
STANFORD
TYMSHARE
SUMEX
NPS
HAWAII
ACCAT
NOSC
UCLA
AFSD
CIT
ISI27
USC
RAND
ISI52
ISI22
UTAH
DOCB
GWC
YUMA
AFWL
WSMR
COLLINS
TEXAS
ANL
DTI
STLA
SCOTT
WPAFB
GUNTER
EGLIN
ROBINS
BRAGG
LINCOLN
MIT44
MIT6
CCA
AFGL
RCC5
RCC49
RCC7
NYU
CORADCOM
DEC
BBN40
BBN63
BBN 72
RADC
CMU
HARVARD
DARCOM
ABERDEEN
ANDRW
NRL
DCEC
NBS
NSA
NORSAR
SDAC
MITRE
ARPA
PENTAGON
LONDON



Wikipedia

WIKIPEDIA

The Free Encyclopedia

Sociedade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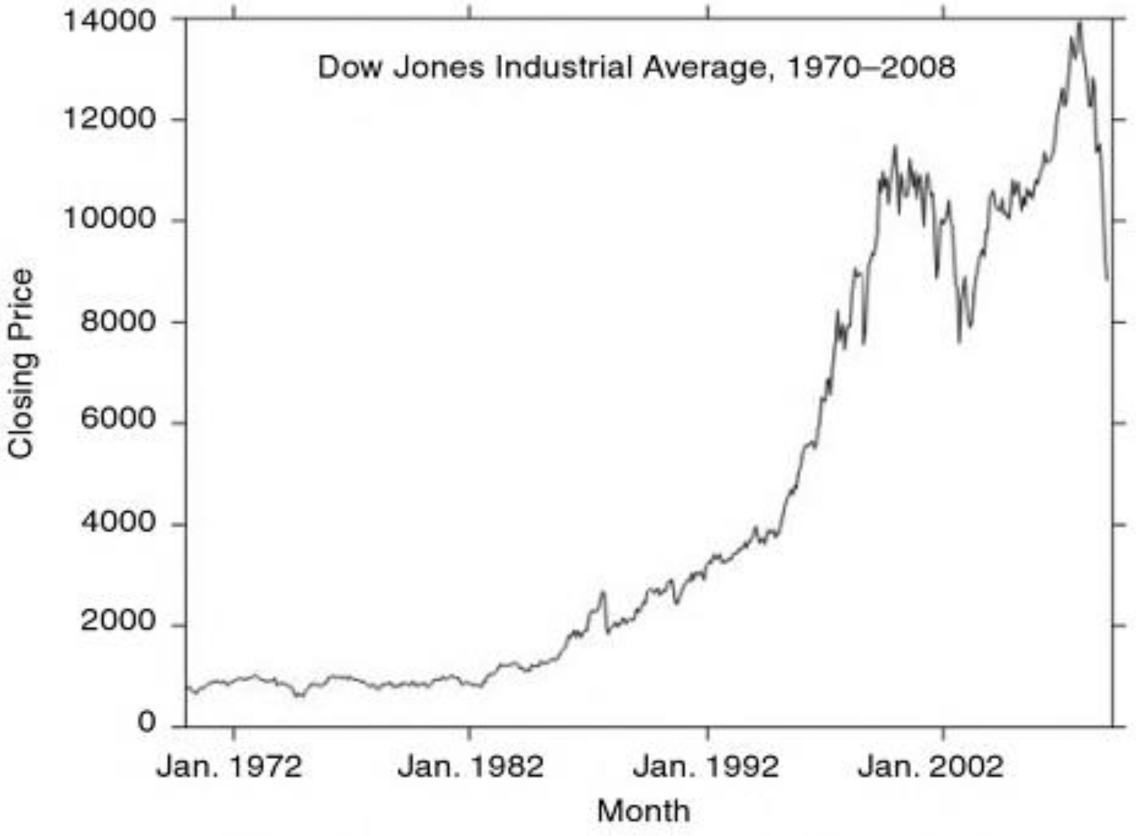
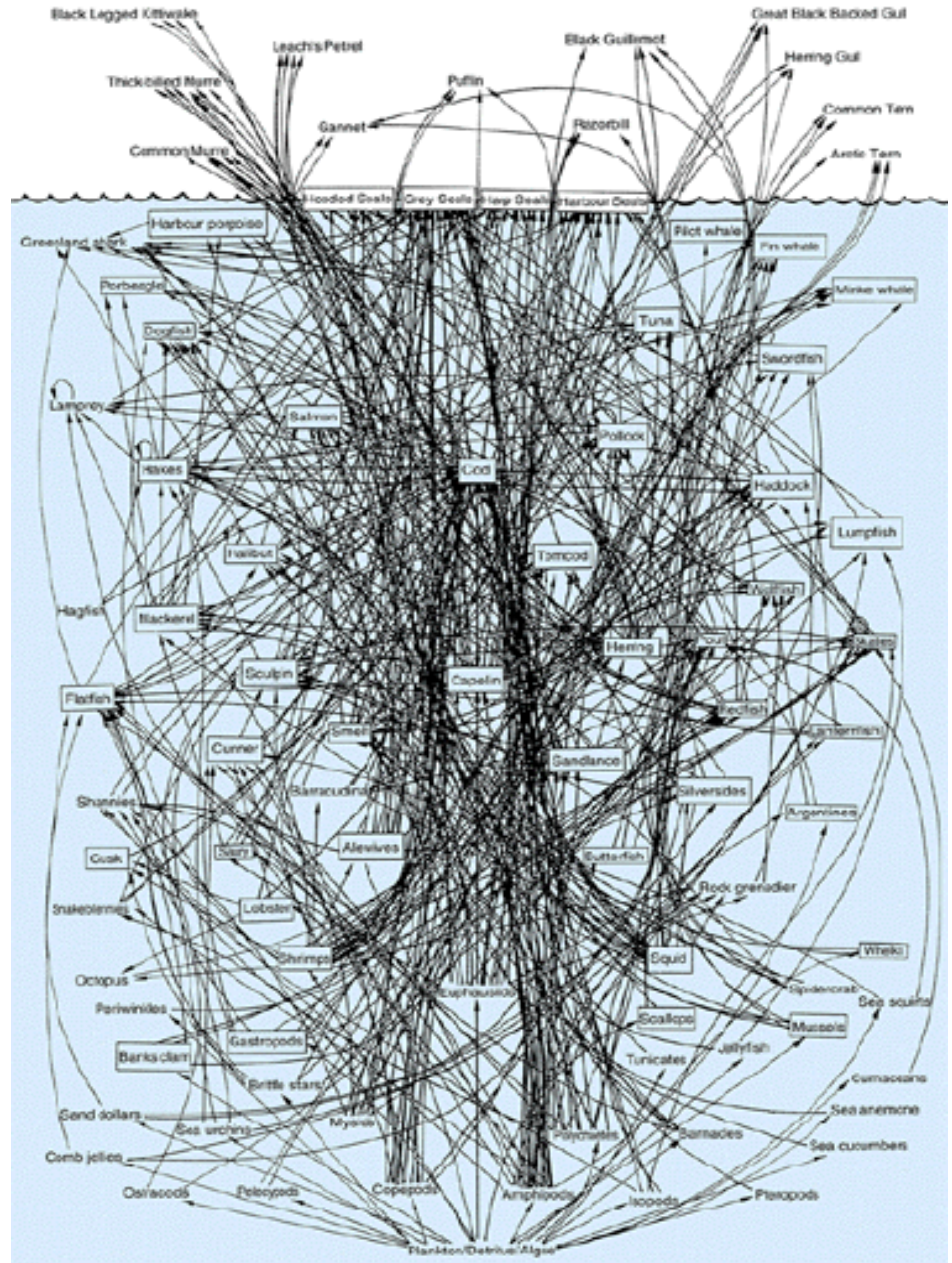
Mercado financeiro



Complexity: a guided tour, M. Mitchell

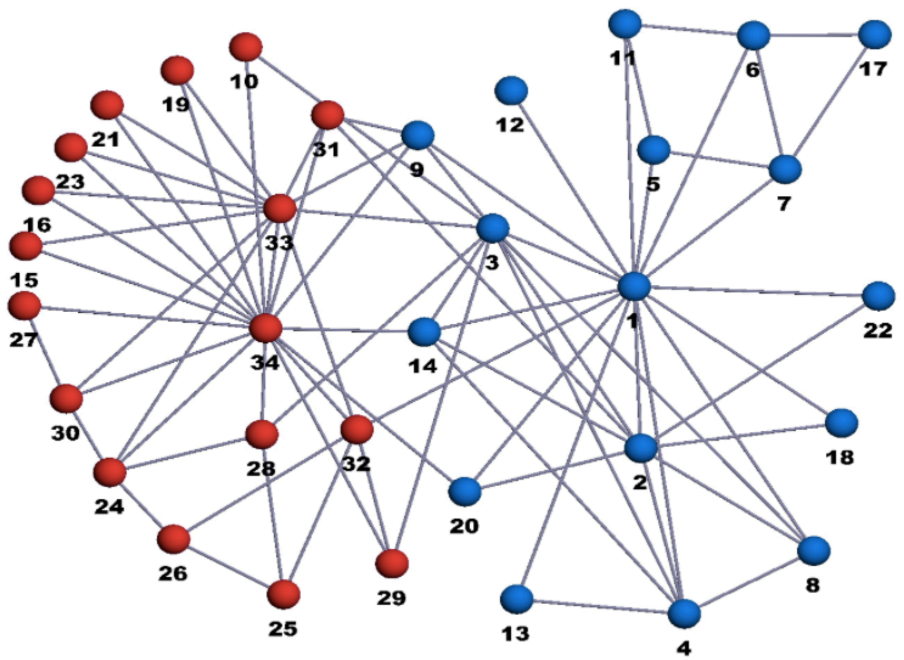
Food webs



A simplified food web for the Northwest Atlantic

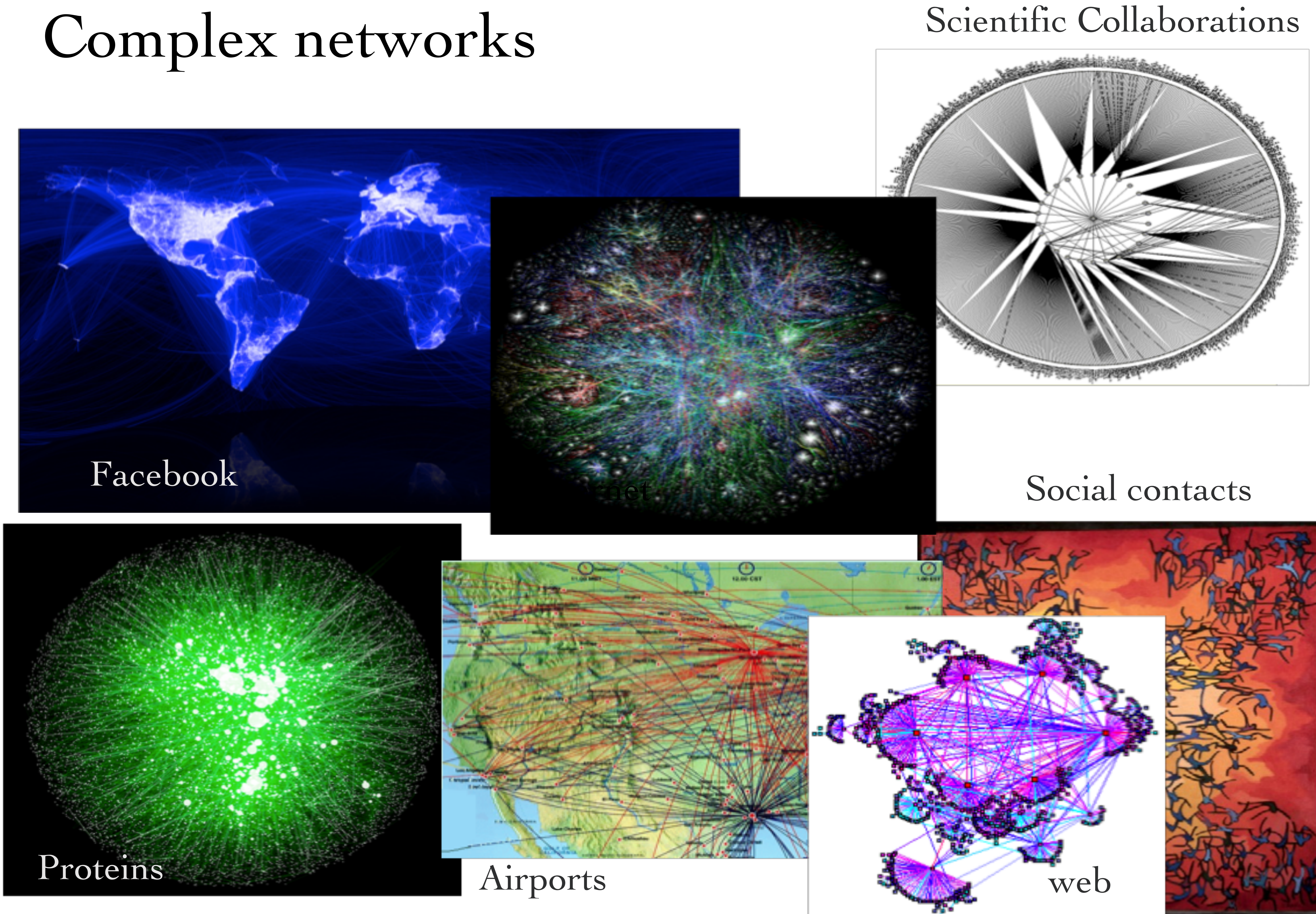
O que esses sistemas tem em comum?

Behind each system studied in complexity there is an intricate wiring diagram, or a **network**, that defines the interactions between the component.



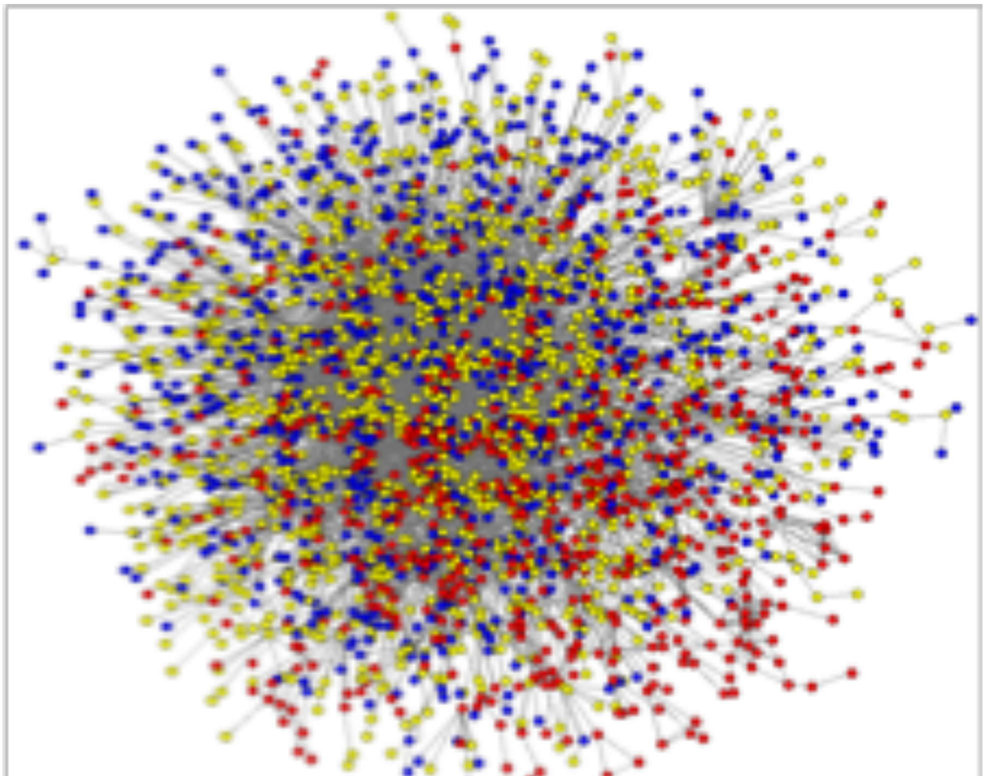
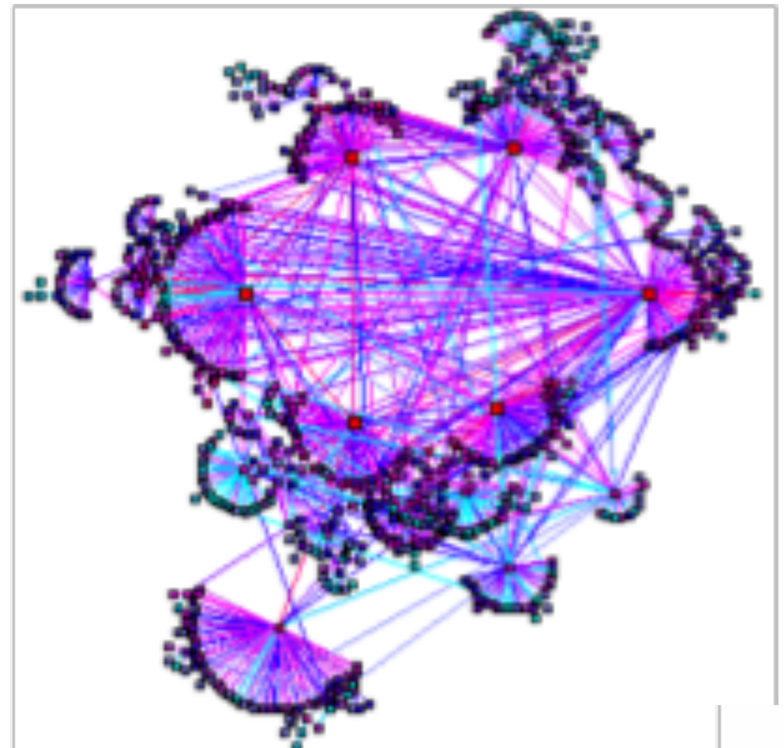
Complex Networks

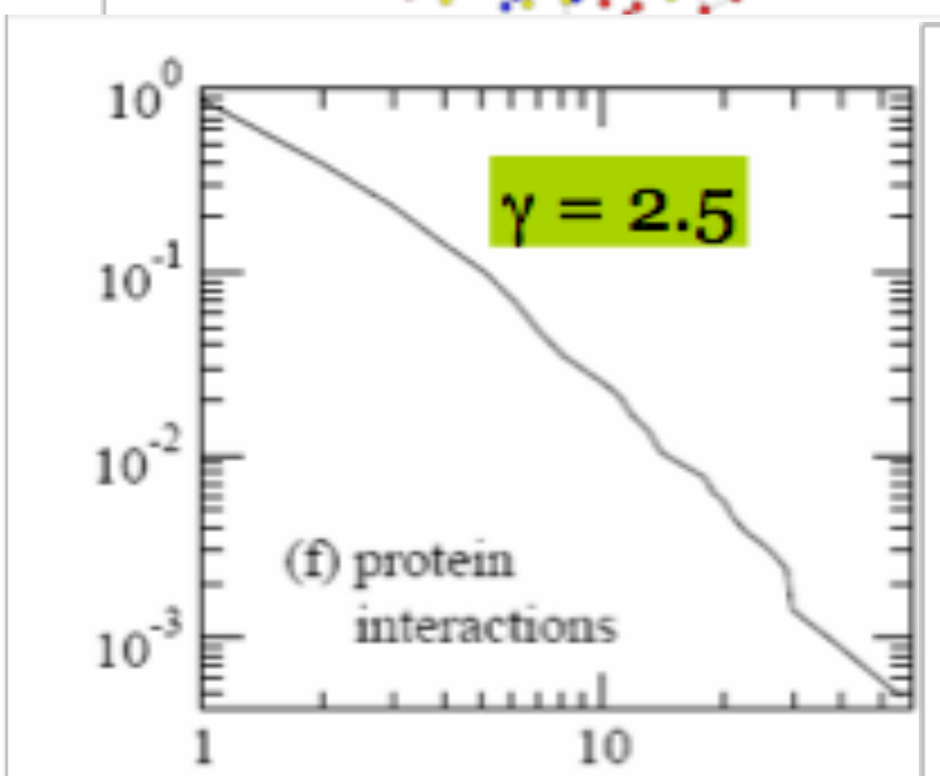
Complex networks

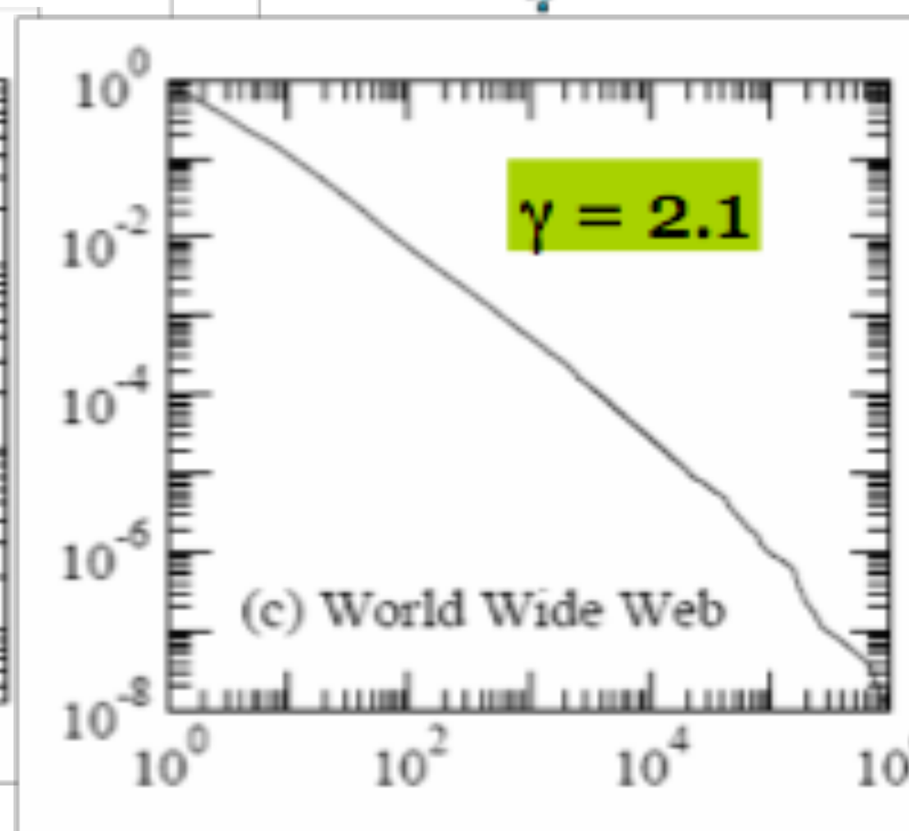


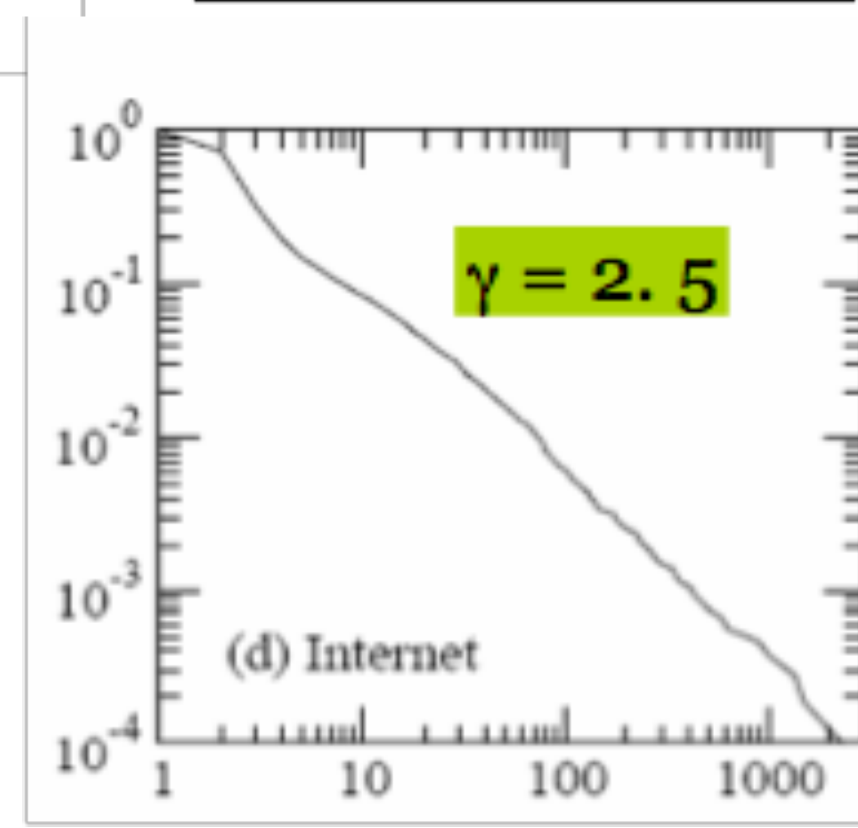
Networks are heterogeneous

$$P(k) \approx k^{-\gamma}$$





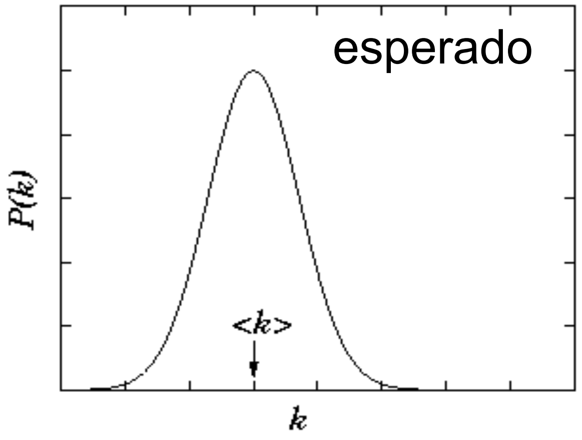


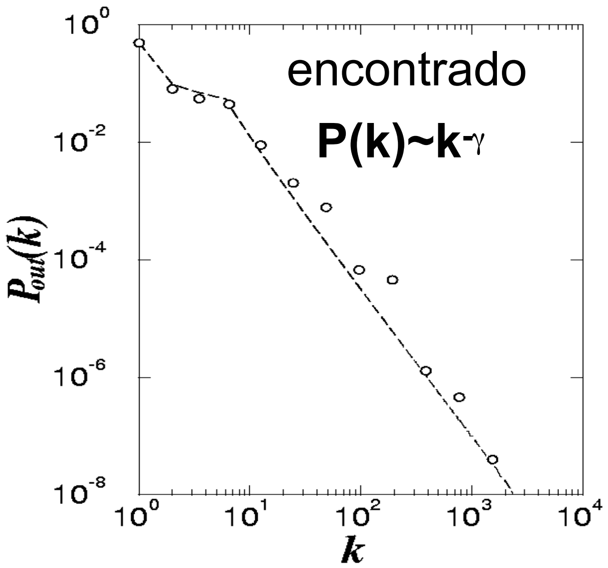
Newman, Siam, 2003

Redes complexas: Primeiras descobertas

World Wide Web (1999)

Vértices: WWW documentos
Links: URL links
Mais de 3 bilhões de páginas





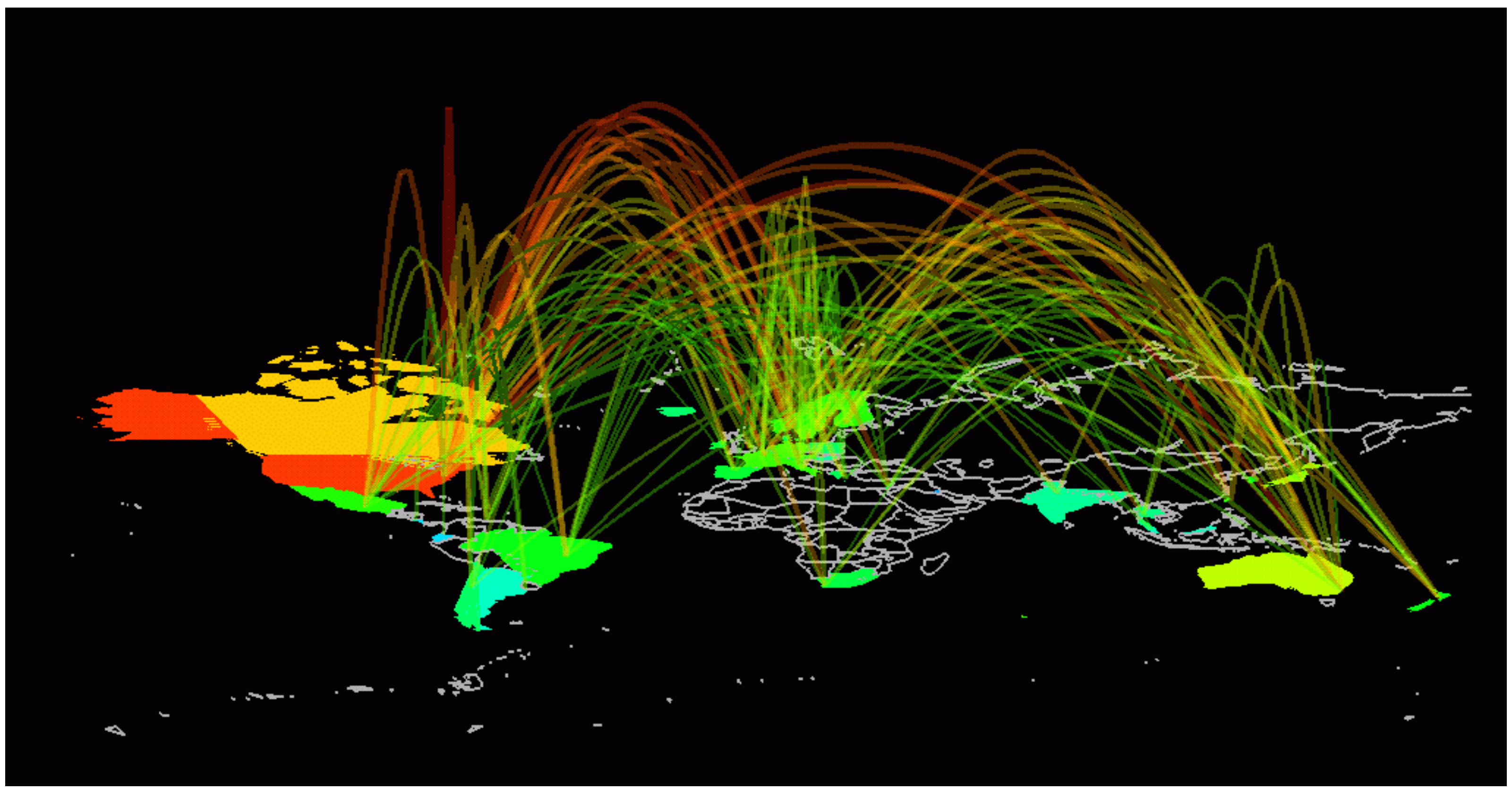
http://www.visualcomplexity.com/vc/

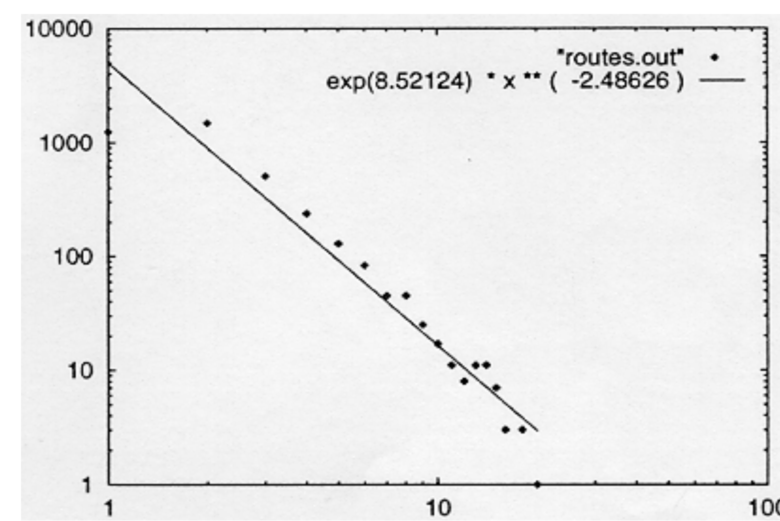
*R. Albert, H. Jeong, A-L Barabasi, Nature, **401** 130 (1999).*

Redes complexas: Primeiras descobertas

Internet (1999)

***Vértices**: computadore,roteadores, sistemas autônomos*
***Links**:   conexões físicas*

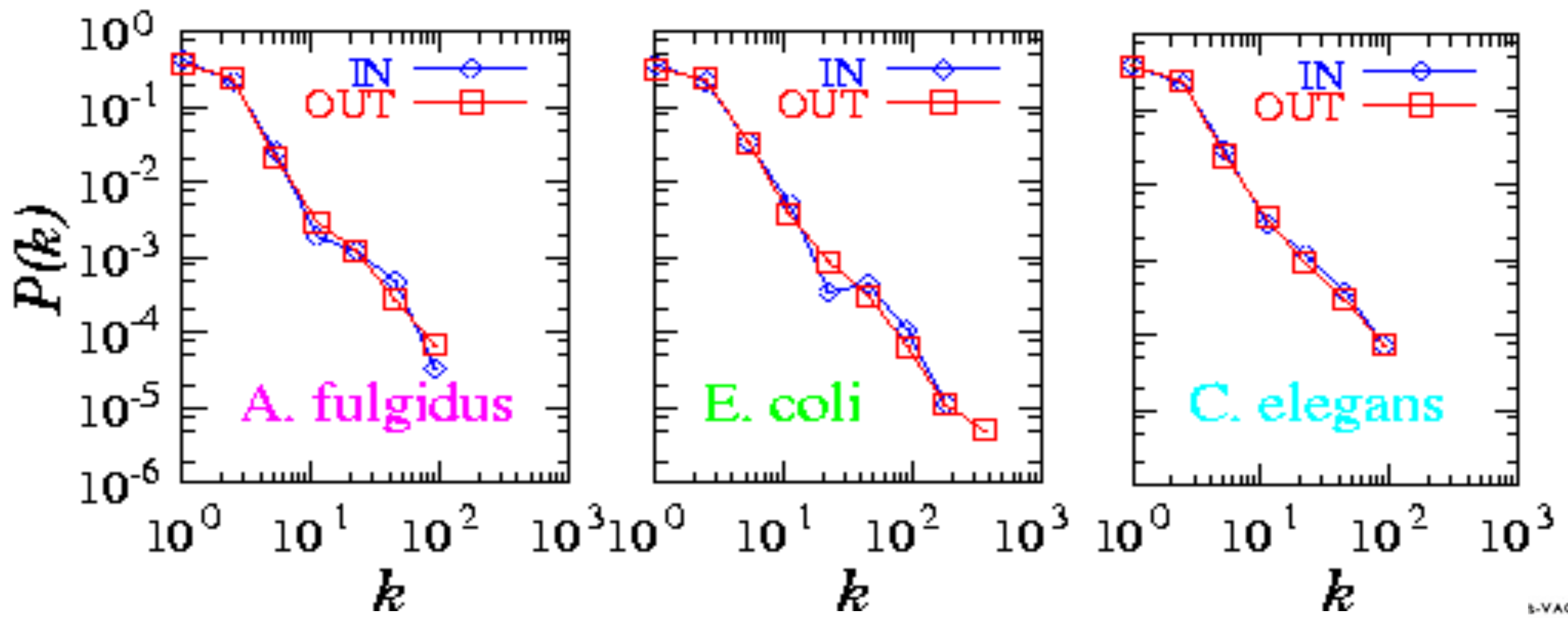




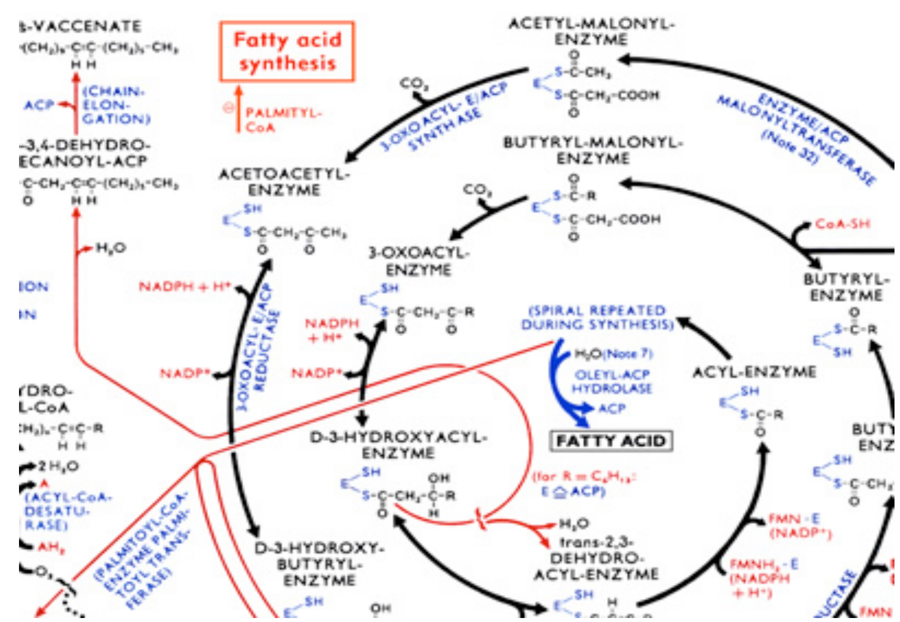
M Faloutsos, P Faloutsos, C Faloutsos - Comput. Commun. Rev, 1999

Redes complexas: Primeiras descobertas

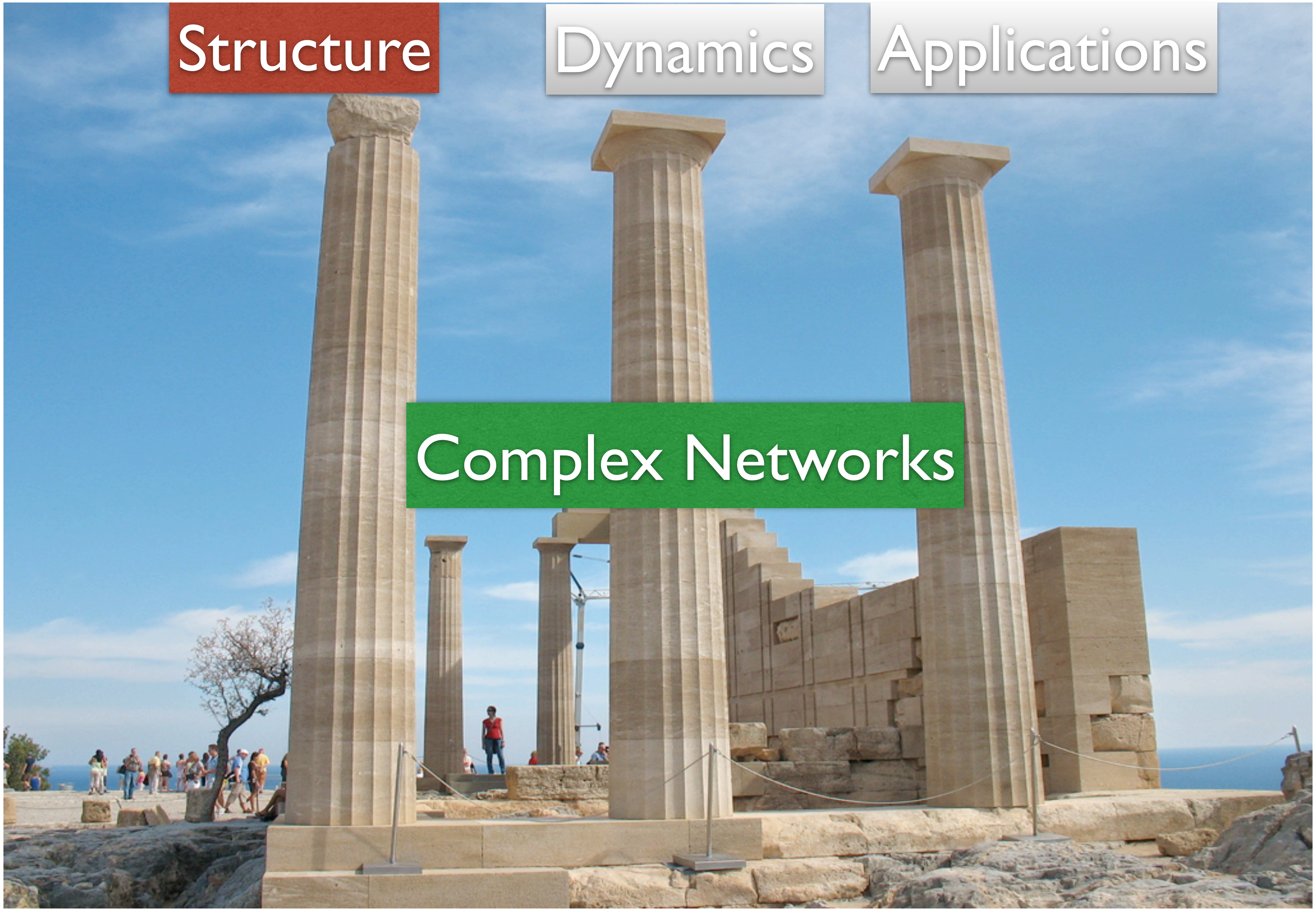
Redes metabólicas



"Organisms from all three domains of life are **scale-free** networks!"



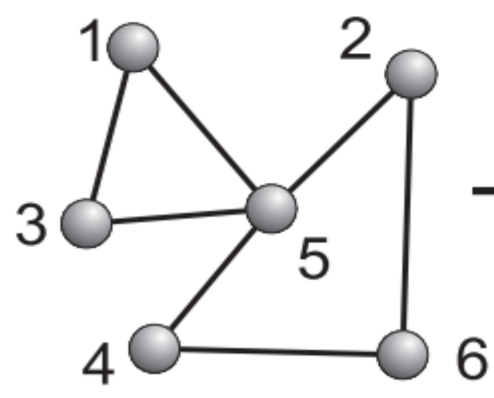
H. Jeong, B. Tombor, R. Albert, Z.N. Oltvai, and A.L. Barabasi, Nature, **407** 651 (2000)


Structure
Dynamics
Applications
Complex Networks

Representação

- **Matriz de adjacências:**

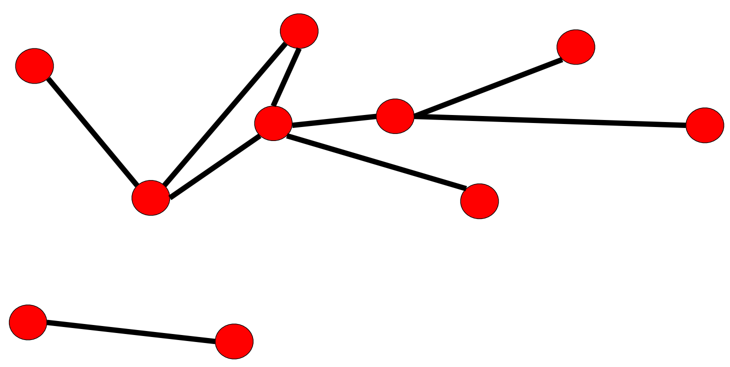
  a_{ij} = 1 se i está ligado com j e 0 caso contrário.

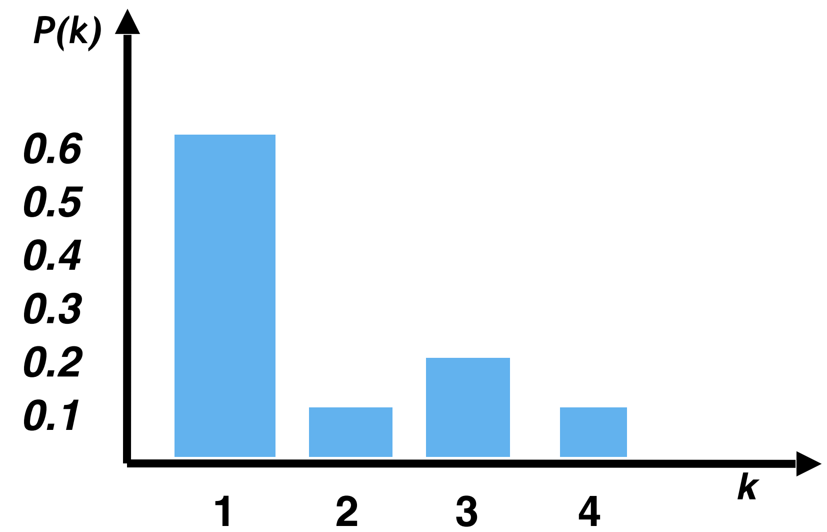


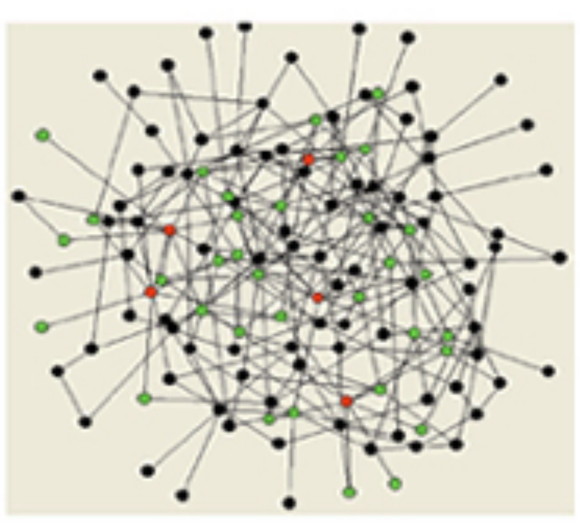
$$\rightarrow A = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

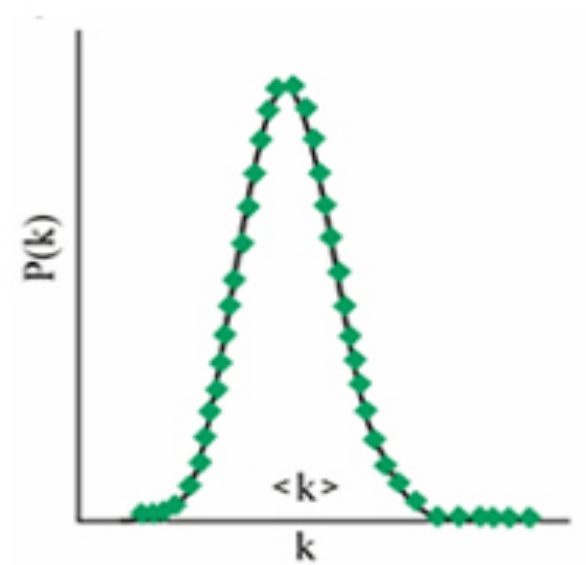
Distribuição do número de conexões

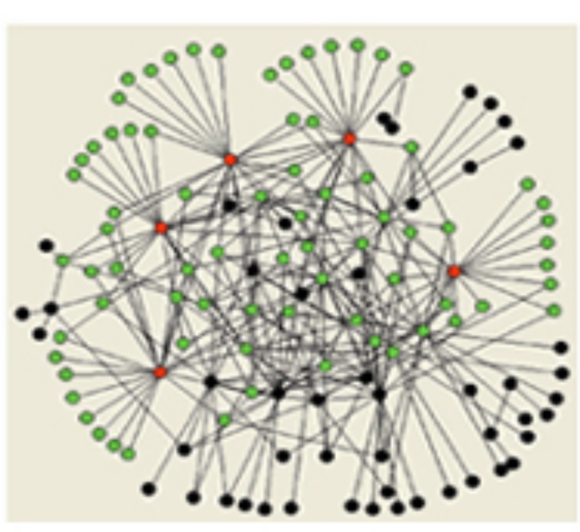
P(k) : probabilidade de que um vértice escolhido tenha grau k

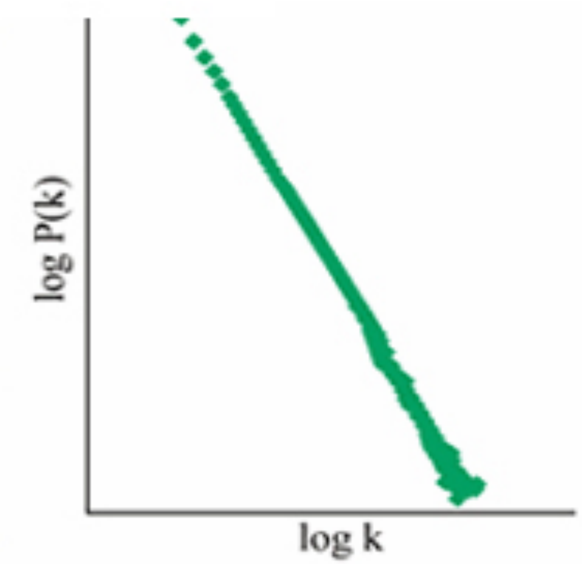






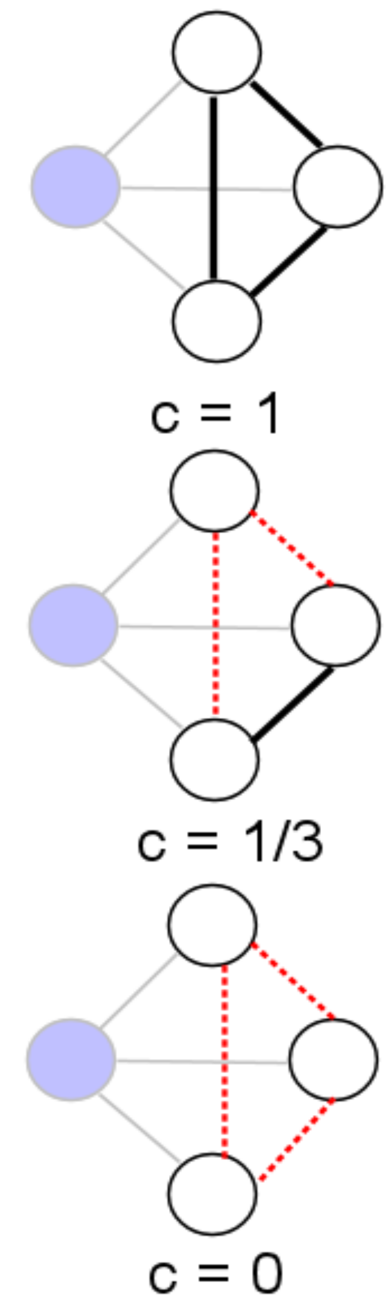




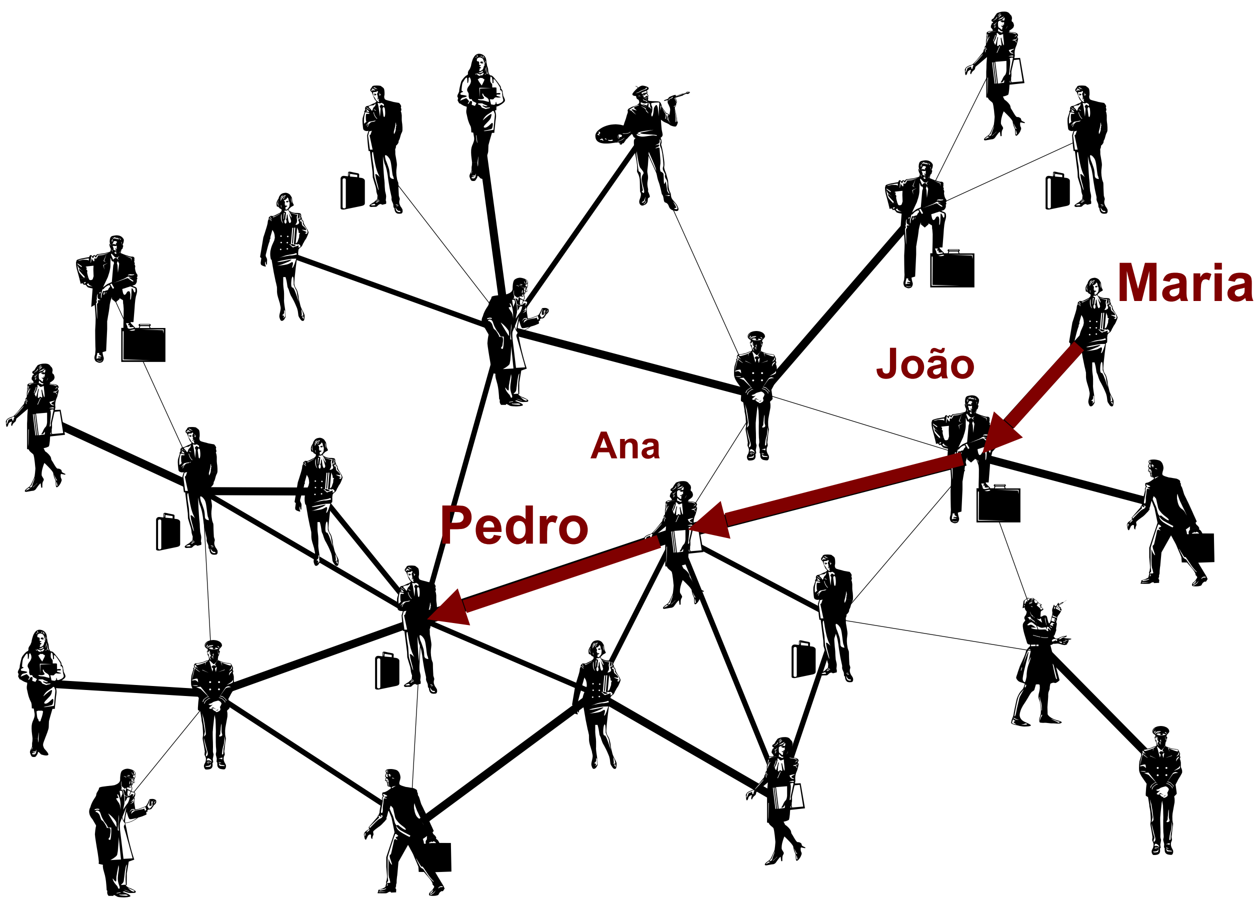


- **Coeficiente de aglomeração**

$$cc_i = \frac{2e_i}{k_i(k_i-1)} = \frac{\sum_{j=1}^{N}\sum_{m=1}^{N} a_{ij}a_{jm}a_{mi}}{k_i(k_i-1)}.$$



Distância

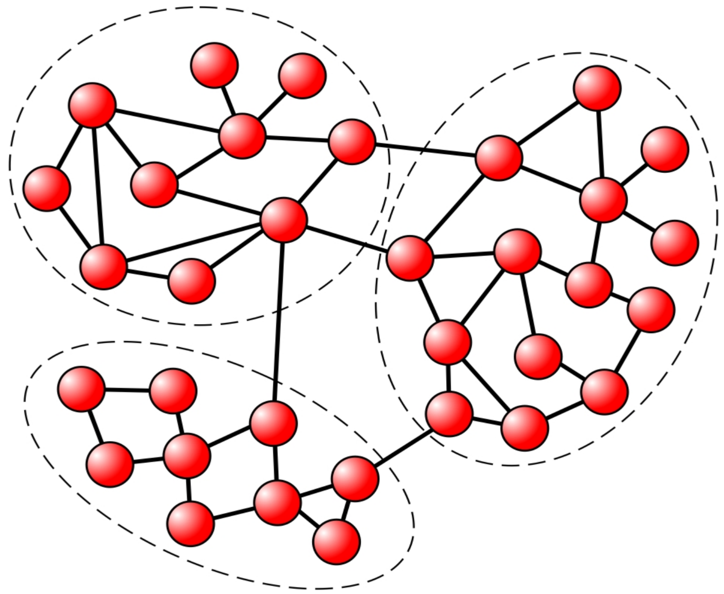


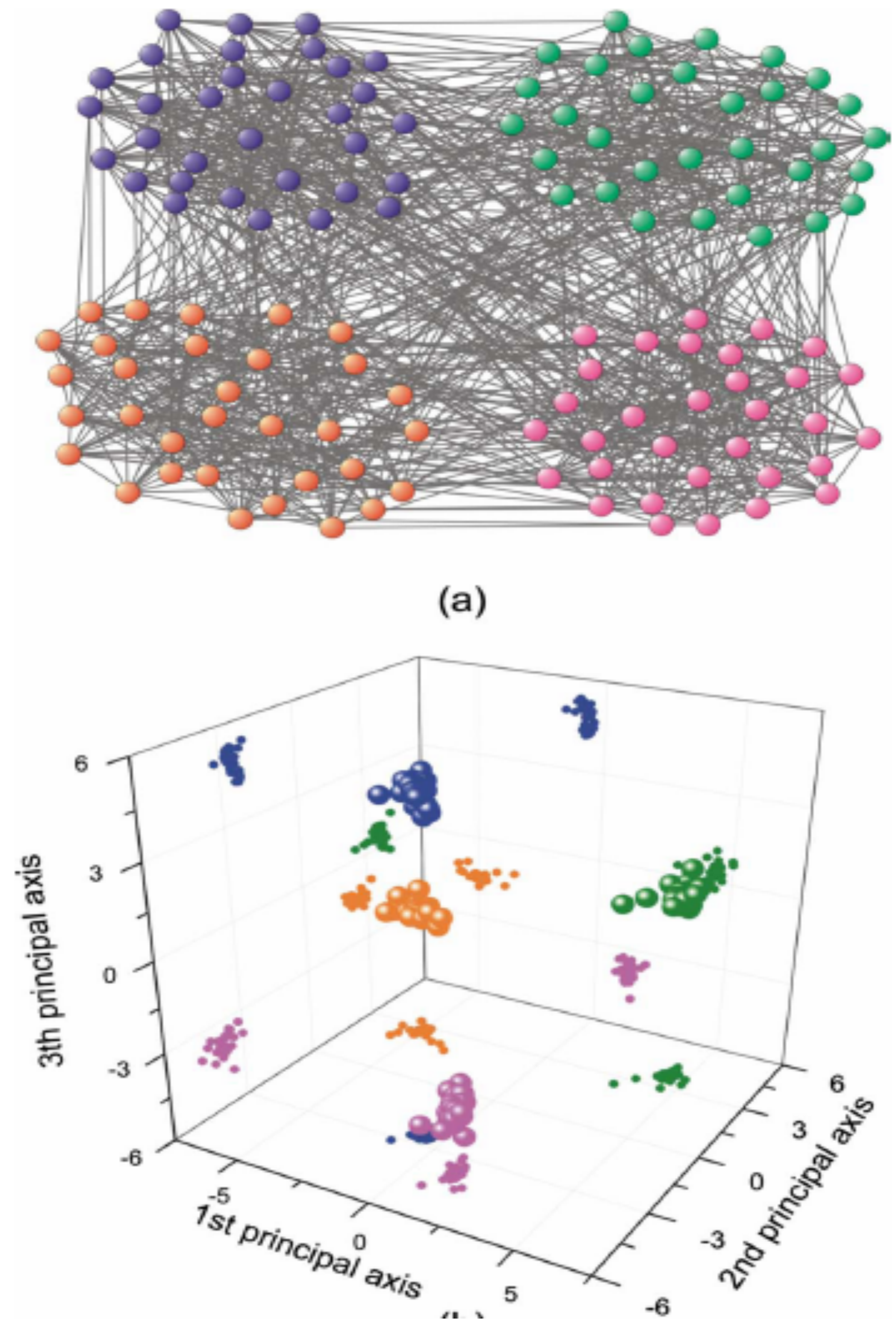
Distância = 3

Sociedade:
Six degrees
S. Milgram 1967

WWW:
19 degrees
Albert et al. 1999

Comunidades



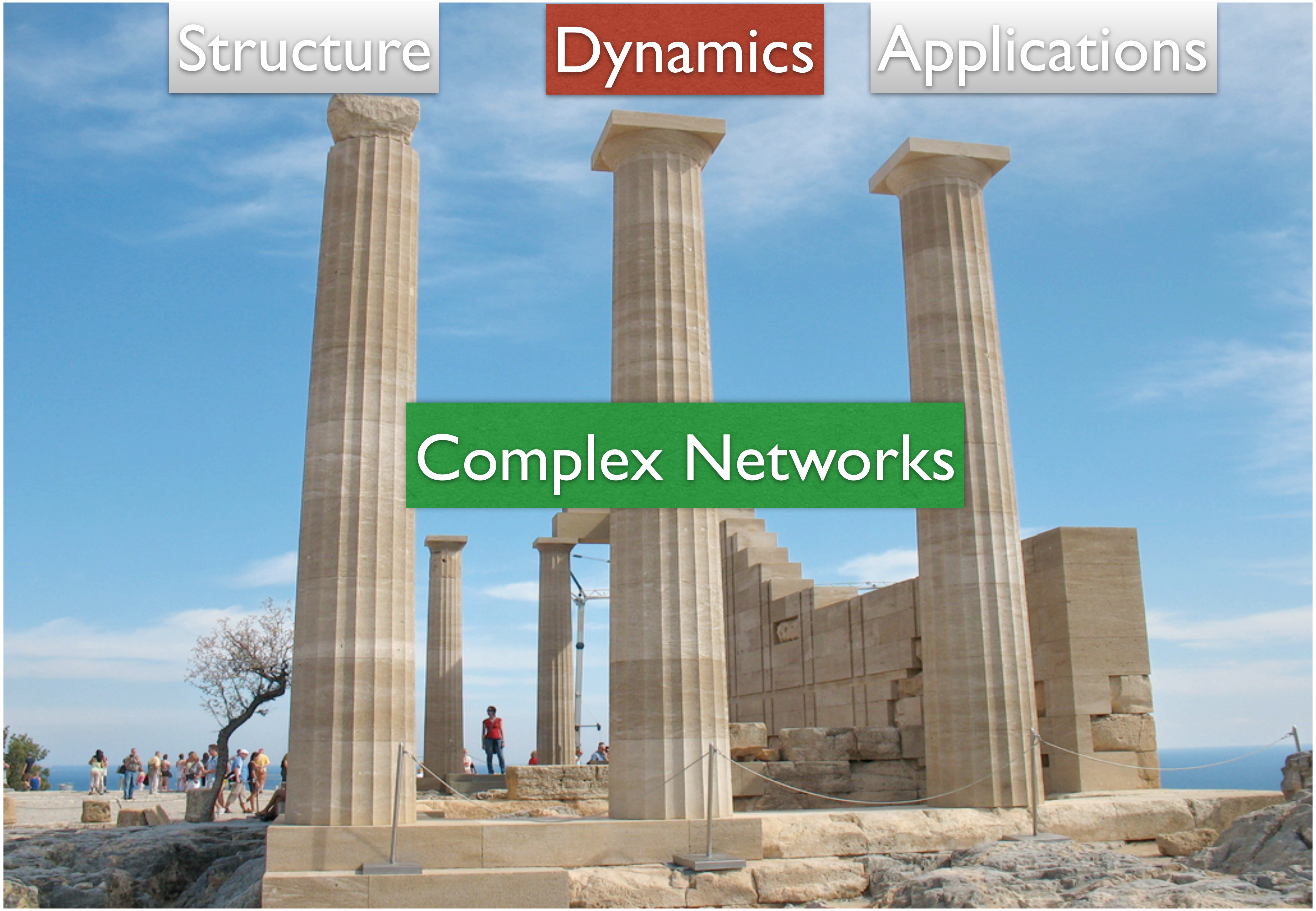


Medidas

- **Medidas fractais,**
- **Medidas espectrais,**
- **Medidas de entropia,**
- **Medidas de centralidade,**
- **Medidas de subgrafos,**
- **...**

Measurement	Symbol
Mean geodesic distance	ℓ
Global efficiency	E
Harmonic mean distance	h
Vulnerability	V
Network clustering coefficient	C and $\widetilde{C}$
Weighted clustering coefficient	C^w
Cyclic coefficient	Θ
Maximum degree	k_{max}
Mean degree of the neighbors	$k_{\text{nn}}(k)$
Degree-degree correlation coefficient	r
Assortativity coefficient	$\widetilde{\mathbb{Q}}, \mathbb{Q}$
Bipartivity degree	b and β
Degree Distribution entropy	$H(i)$
Average search information	$\mathcal{S}$
Access information	$\mathcal{A}_i$
Hide information	$\mathcal{H}_i$
Target entropy	$\mathcal{T}$
Road entropy	$\mathcal{R}$
Betweenness centrality	B_i
Central point dominance	CPD
lth moment	M_l
Modularity	Q
Participation coefficient	P_i
z-score	z_i
Significance profile	SP_i
Subgraph centrality	SC
Hierarchical clustering coefficient	C_{rs}
Convergence ratio	$cv_d(i)$
Divergence ratio	$dv_d(i)$
Edge reciprocity	ϱ and ρ
Matching index of edge (i, j)	μ_{ij}

Costa, Rodrigues, Travieso, Villas Boas. Advances in Physics 2007

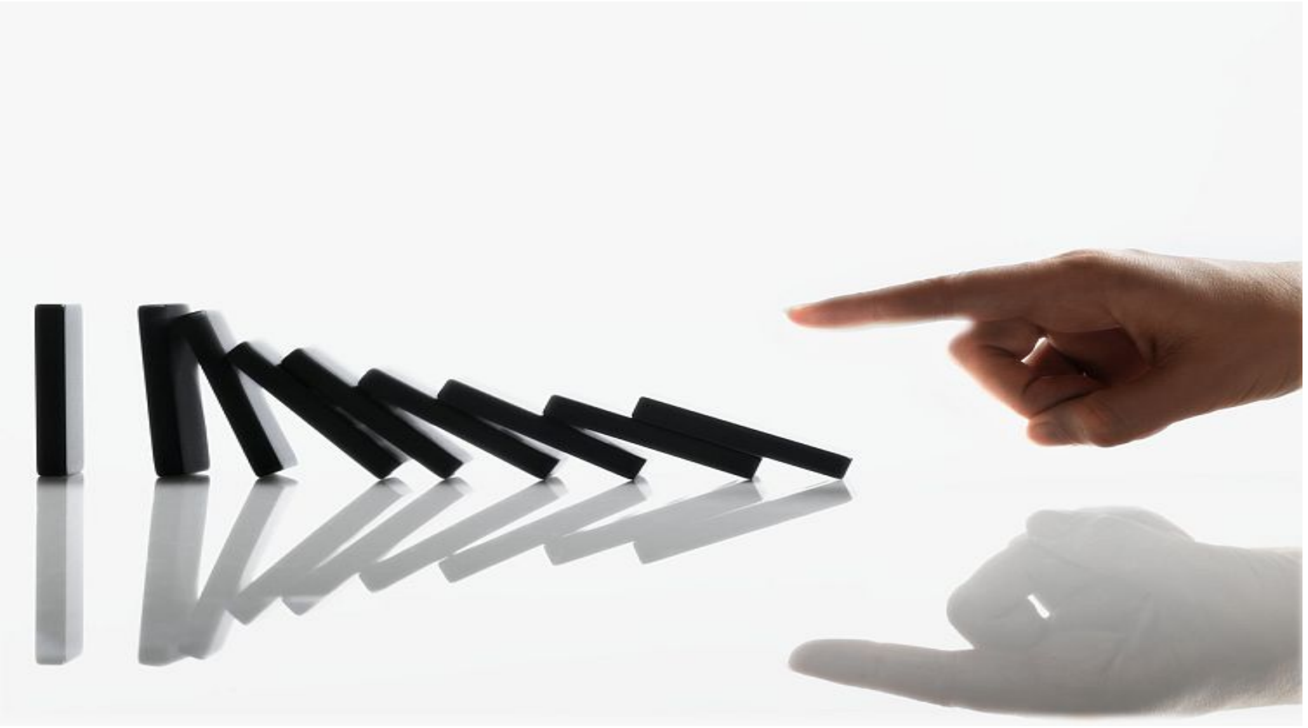

Structure
Dynamics
Applications
Complex Networks

- **Falhas e ataques,**
- **Propagação de epidemias,**
- **Propagação de rumores,**
- **Caminhadas aleatórias,**
- **Sincronização,**
- **Transporte,**
- **Dinâmicas sociais,**
- **Transmissão de sinais.**

....

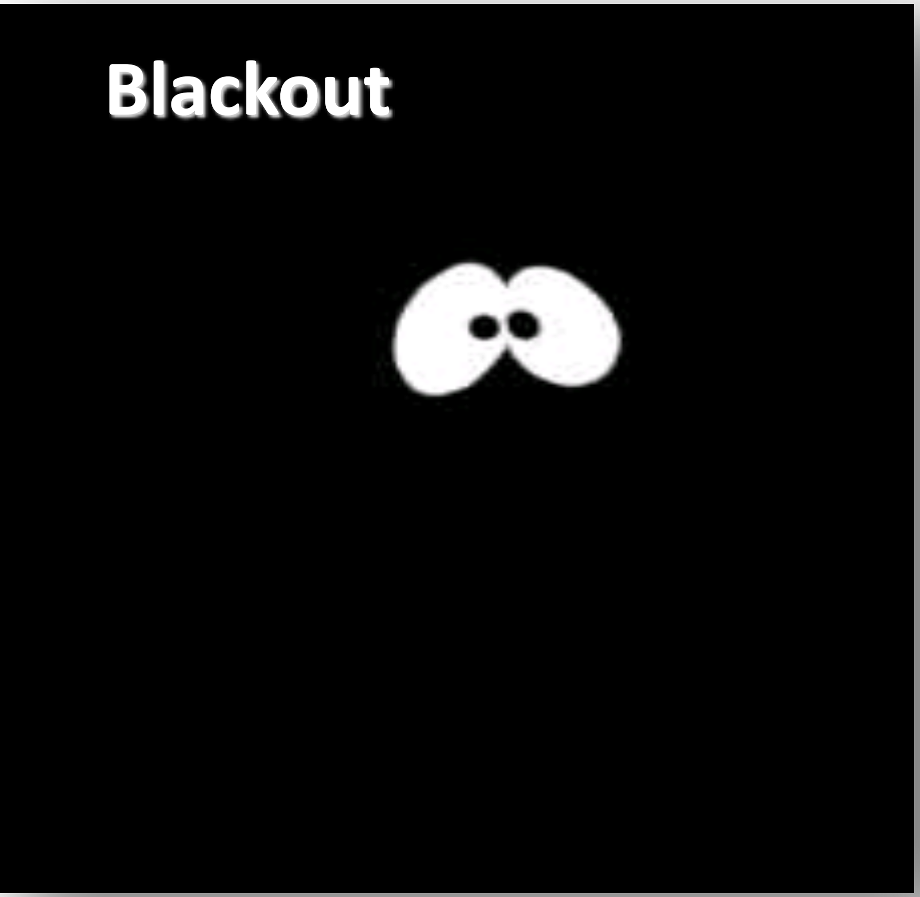
Falhas em cascata

Potentially large events triggered by small initial shocks



- **Information cascades**
  social and economic systems
  diffusion of innovations
- **Cascading failures**
  infrastructural networks
  complex organizations

Falhas em cascata



Flows of physical quantities

•congestions

•instabilities

•Overloads

Cascades depend on

- Structure of the network
- Properties of the flow
- Properties of the net elements
- Breakdown mechanism

Falhas em cascata

1 - Cada vértice tem um carga inicial.
2 - Remova um vértice aleatoriamente ou de acordo com alguma propriedade particular.
3 - Calcule a nova carga.
4 - Remova os vértices que excedem a sua carga máxima.
5 - Repita a partir do passo 3 até que não haja mais vértices para serem removidos.

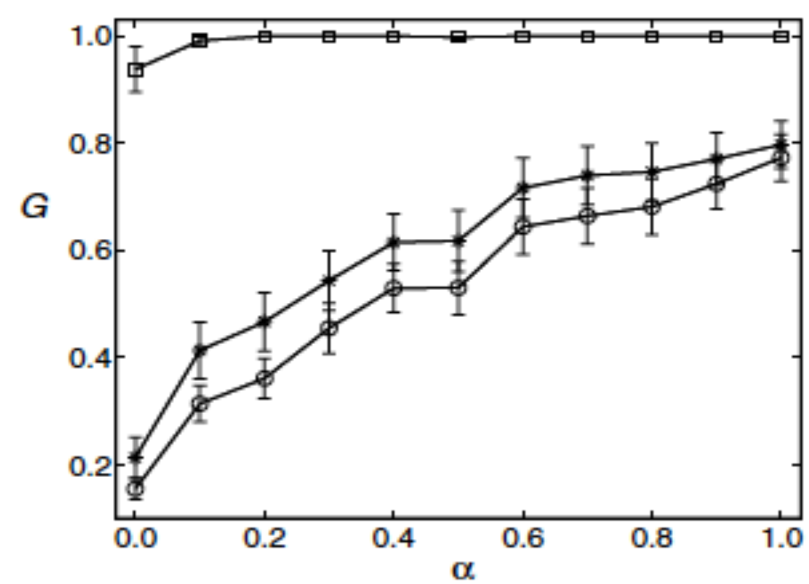


Fig. 2. Cascading failure in scale-free networks with scaling exponent $\gamma = 3$, as triggered by the removal of one node chosen at random (squares), or among those with largest connectivities (stars) or highest loads (circles). Each curve corresponds to the average over 5 triggers and 10 realizations of the network. The error bars represent the standard deviation. The number of nodes in the largest component is $5000 \leq N \leq 5100$.

Epidemic Spreading

430 B.C. Plague of Athens
25% population

1300-1700 Plague
~75-200 million died

1816-1923

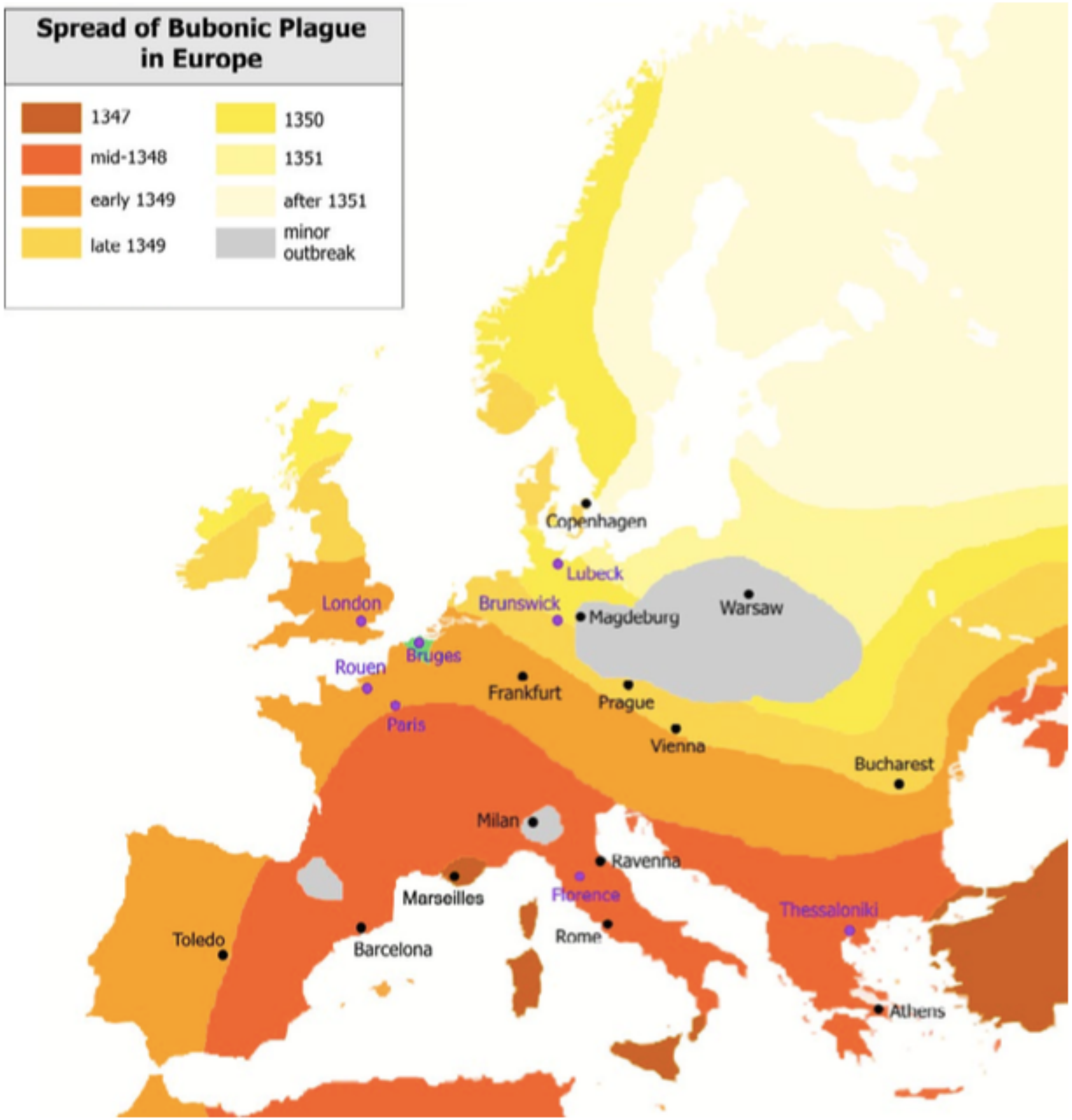
1918-1920 Spanish Flu
20-100 million died

2003 S.A.R.S.
775 deaths

2009 H1N1 (Swine) Flu
18000 deaths

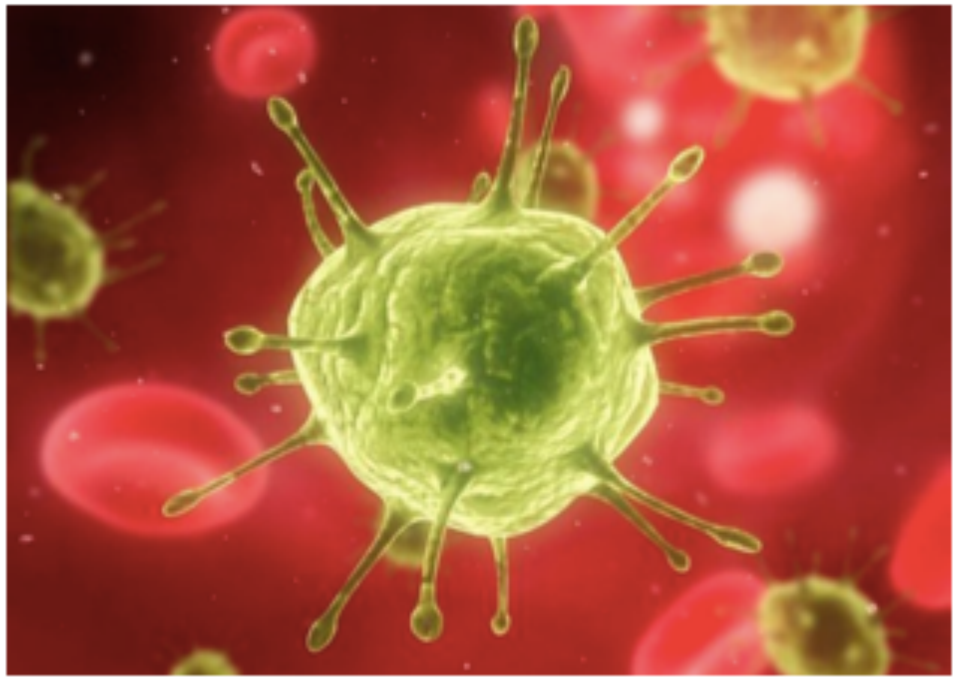
time

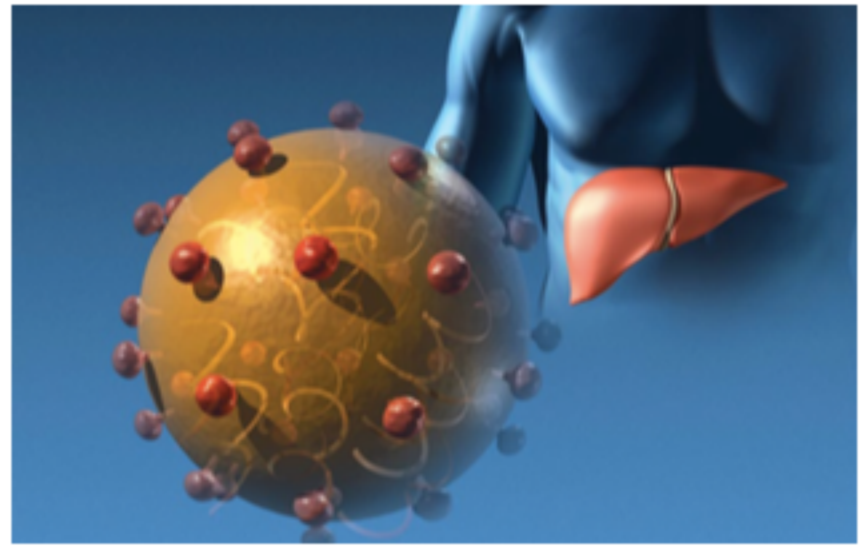
Bubonic Plague

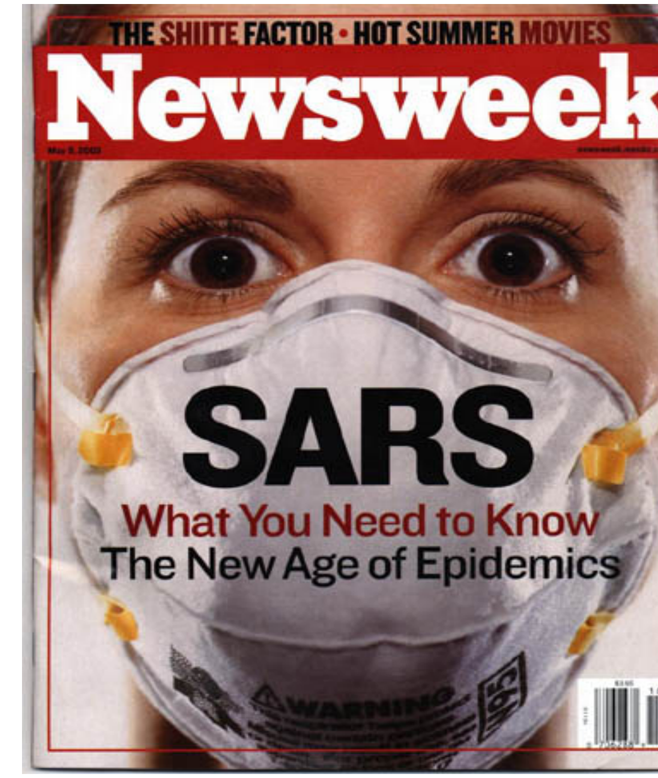


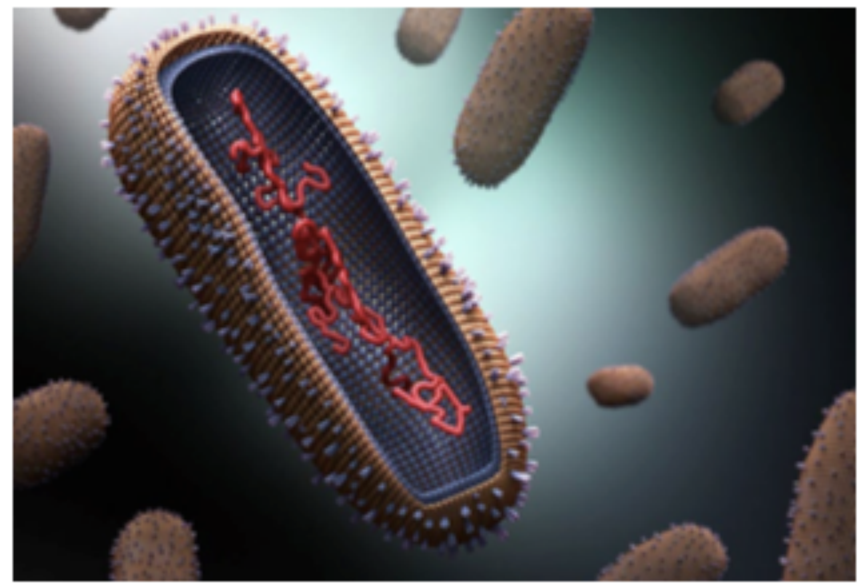
HINI

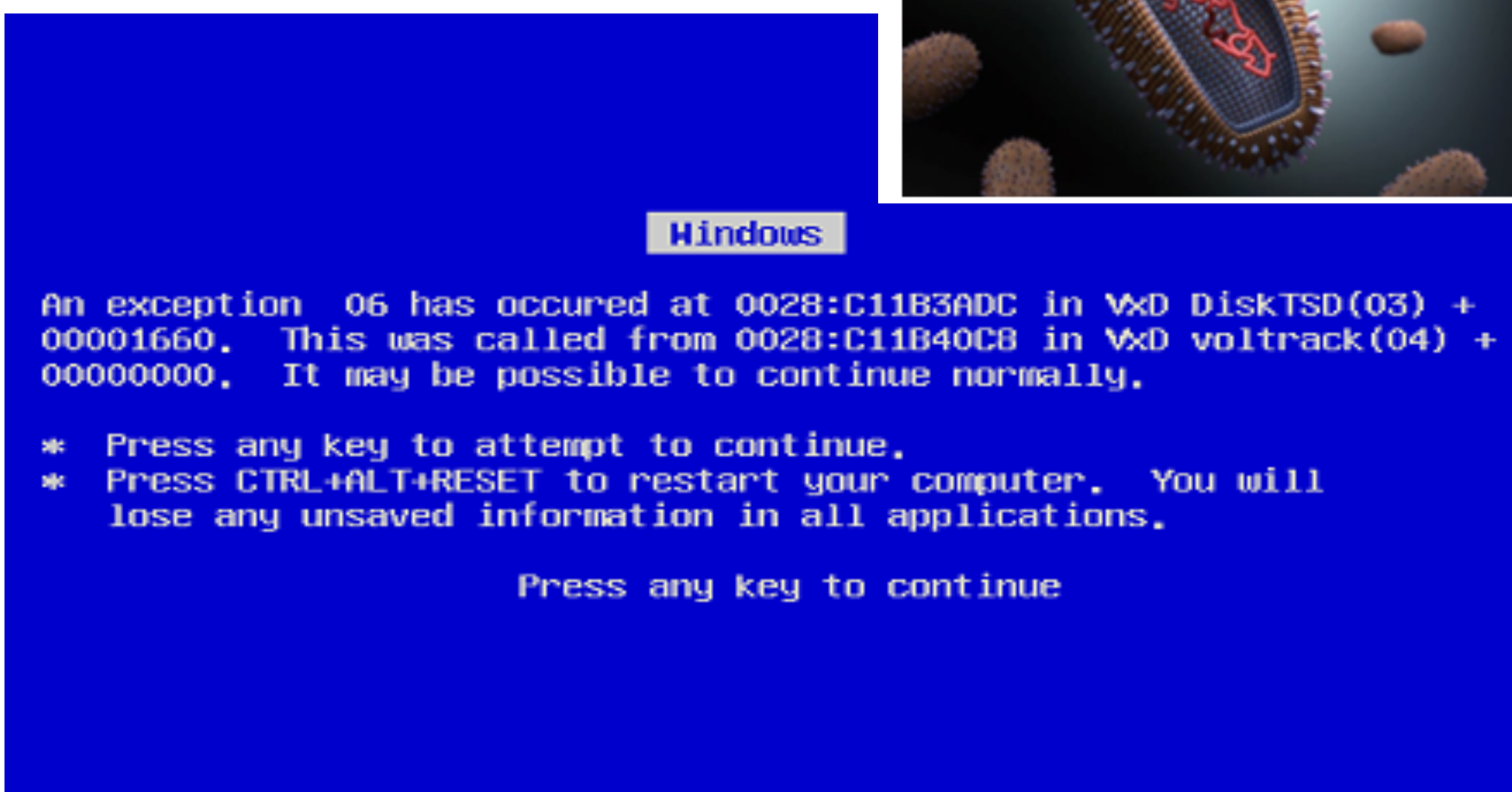
Spreading depends on the network structure!

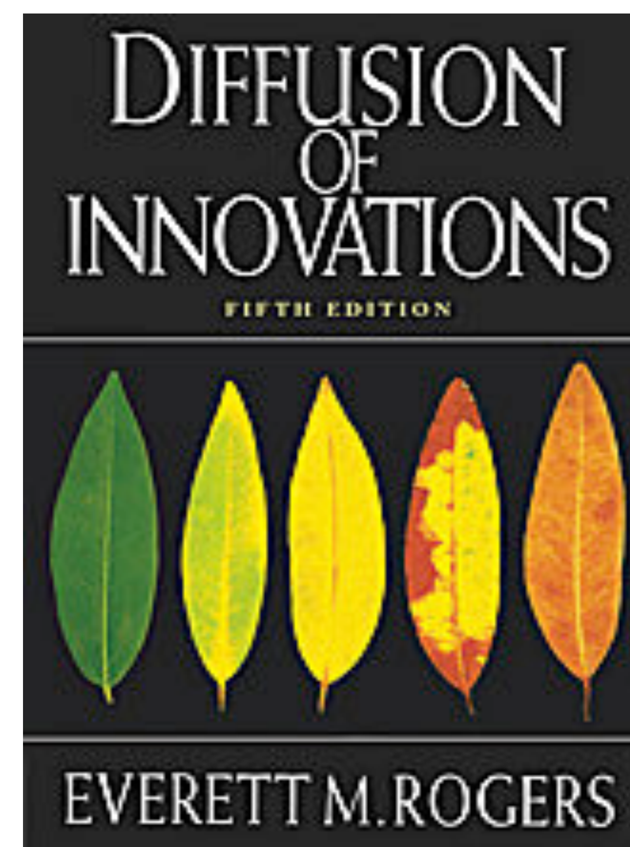





THE SHIITE FACTOR • HOT SUMMER MOVIES
Newsweek
SARS
What You Need to Know
The New Age of Epidemics




Windows
An exception 06 has occured at 0028:C11B3ADC in VxD DiskTSD(03) + 00001660. This was called from 0028:C11B40C8 in VxD voltrack(04) + 00000000. It may be possible to continue normally.
* Press any key to attempt to continue.
* Press CTRL+ALT+RESET to restart your computer. You will lose any unsaved information in all applications.
Press any key to continue


DIFFUSION OF INNOVATIONS
FIFTH EDITION
EVERETT M. ROGERS

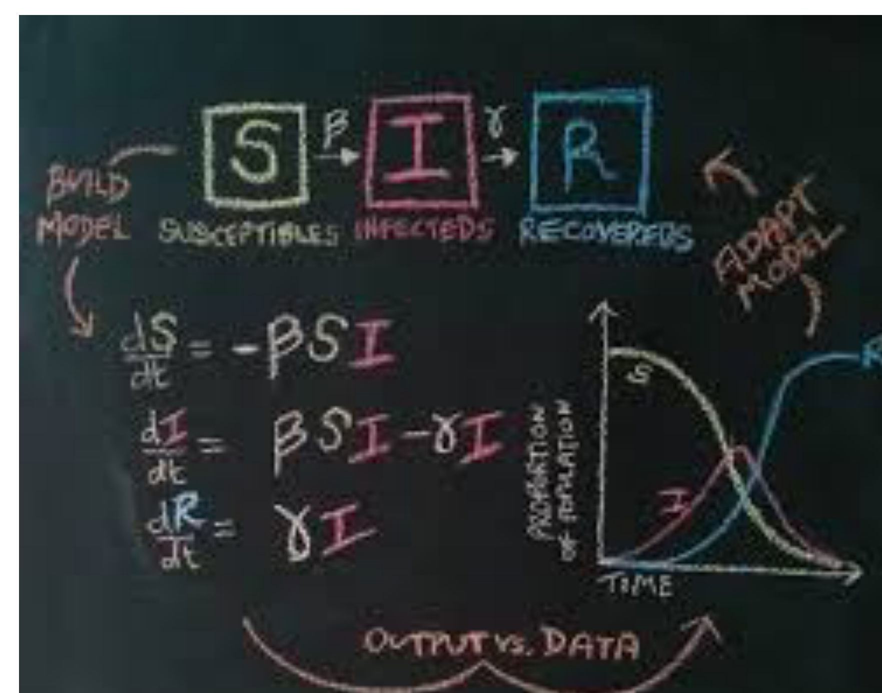
Questions

- Will an infection spread in a network?
- How is the influence of the network on the spreading process?
- How does it evolve over time?
- Can we predict the impact of a disease on a network?
- How to control an outbreak?

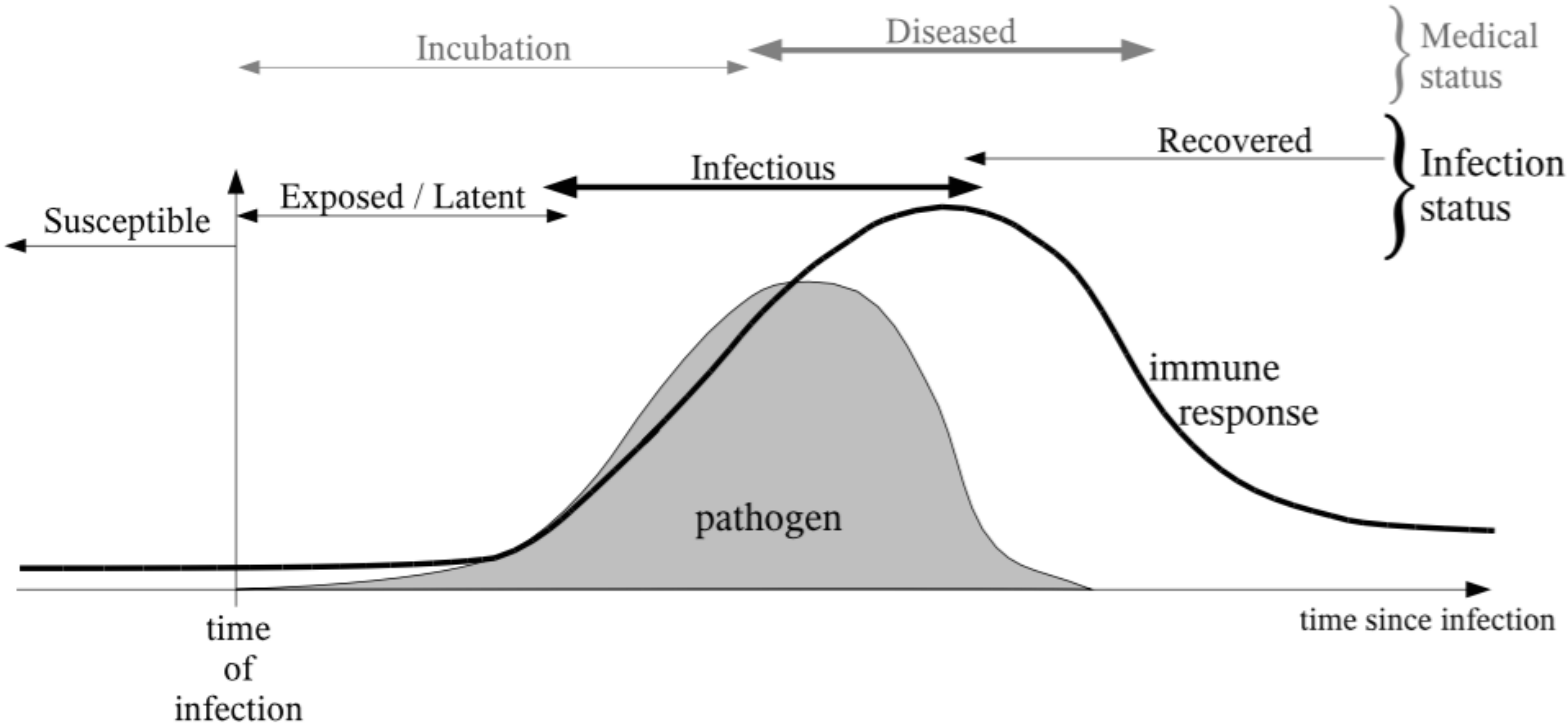
Epidemic Spreading

Prediction, understanding, control:
Mathematical Modeling

- Mathematical modeling of infectious diseases is a tool to investigate the mechanisms for outbreak and spread of diseases and to predict the future course in order to control an epidemic.

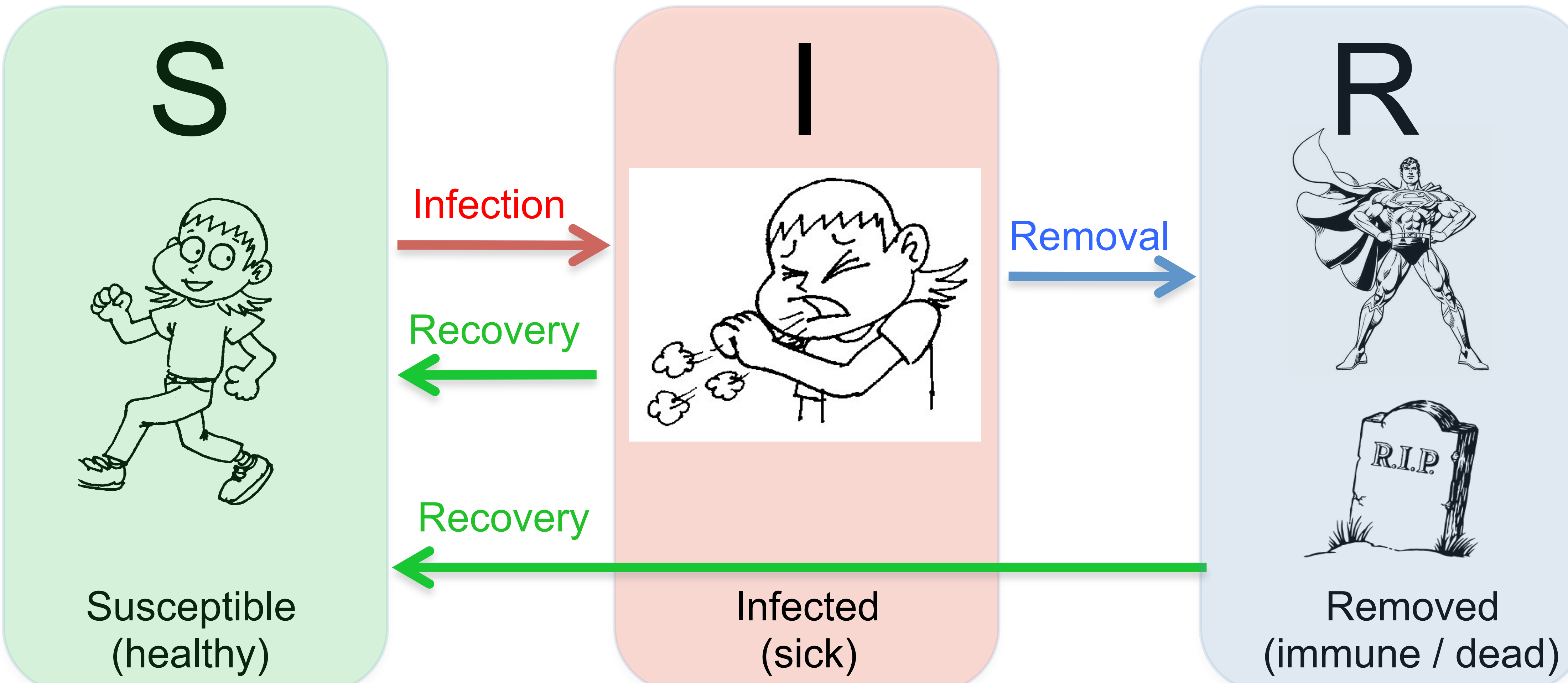


Epidemic Spreading

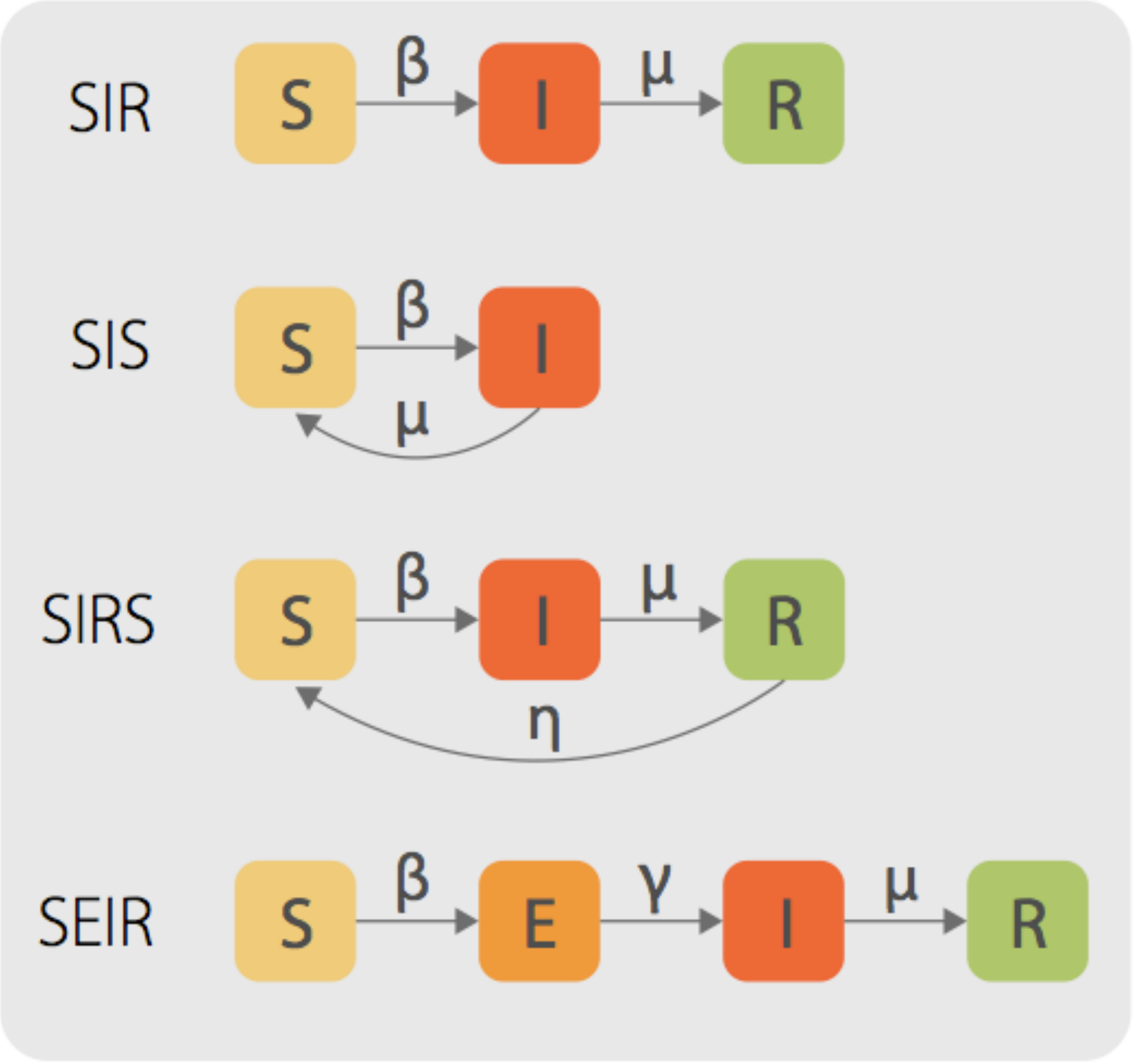


Keeling and Rohani, Modeling Infectious Diseases in humans and animals, Princeton Un. Press

Deterministic compartmental models



Deterministic compartmental models



Pastor-Satorras et al. Reviews of Modern Physics 2014

Deterministic compartmental models: SIR

$$S + I \xrightarrow{\beta} S + S$$
$$S \xrightarrow{\mu} R$$

where

S – the number of susceptibles

I – the number of infectives

R – the number of recoveries

β – contact rate

Deterministic compartmental models: SIR

$$S + I \xrightarrow{\beta} S + S$$
$$S \xrightarrow{\mu} R$$

$$\frac{dS}{dt} = -\beta S I$$
$$\frac{dI}{dt} = \beta S I - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

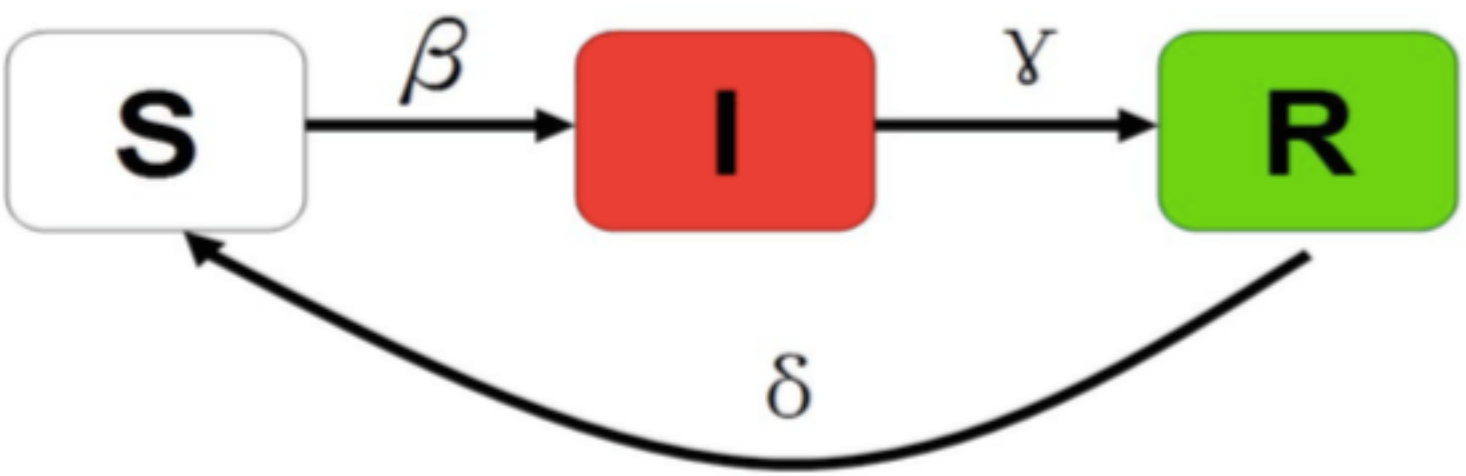
Deterministic compartmental models: SIS

SIS epidemic (disease w/o immunity)

$$\frac{dS}{dt} = -\beta SI + \gamma I$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$

Deterministic compartmental models: SIRS

SIRS epidemic (disease with finite-time immunity)



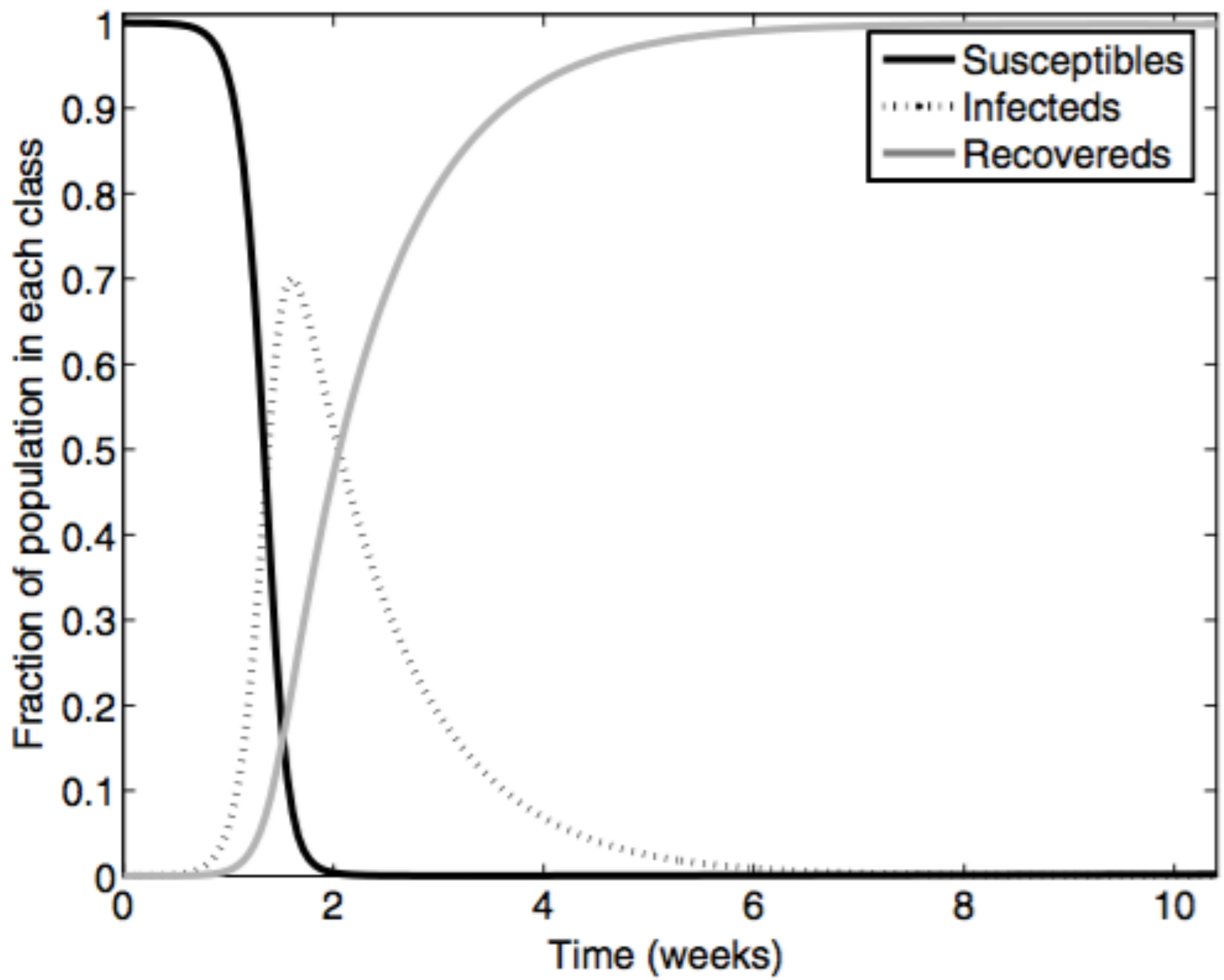
$$\frac{dS}{dt} = -\beta SI + \delta R$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I - \delta R$$

Epidemic Spreading

SIR



Epidemic Spreading

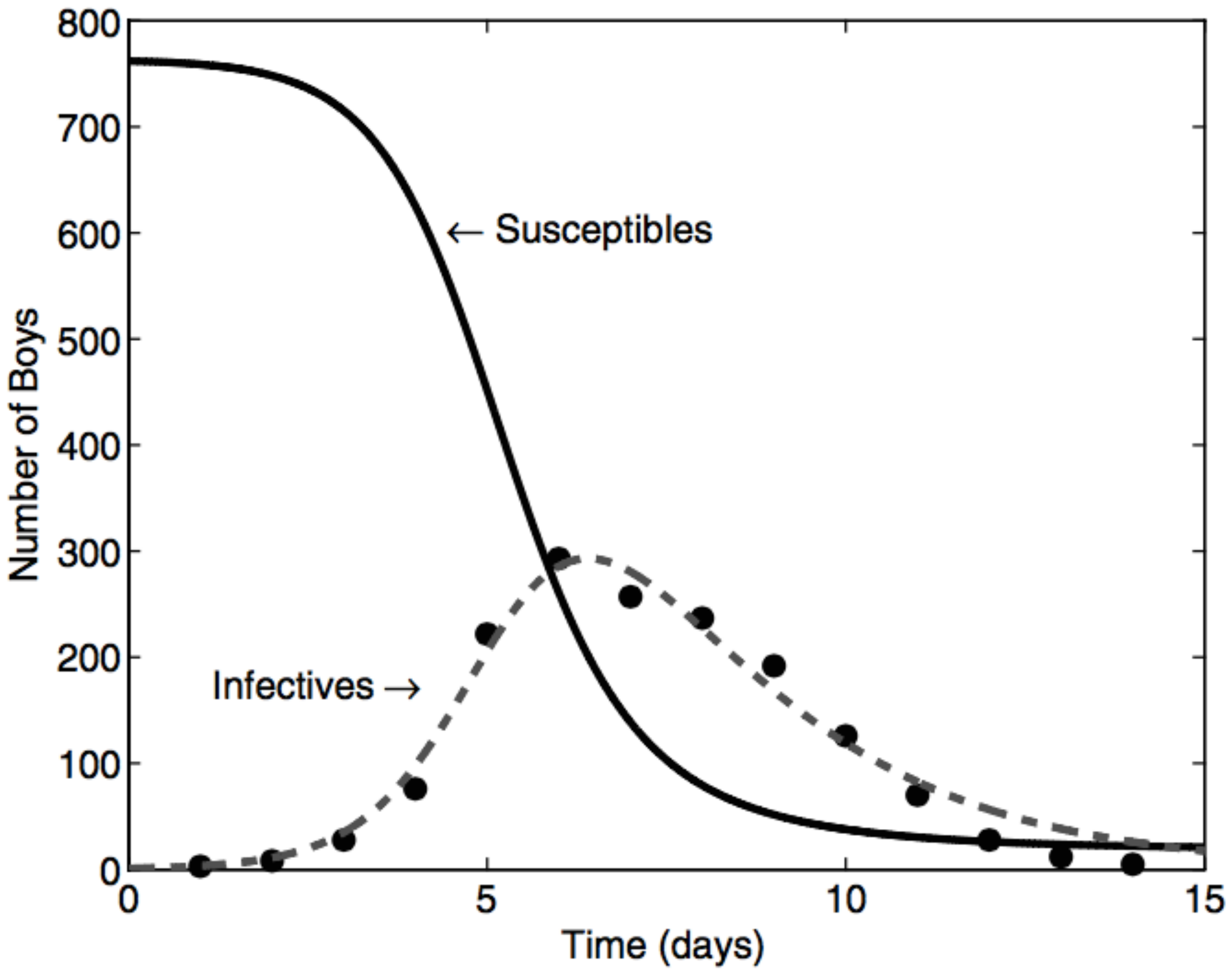


Figure 2.4. The *SIR* dynamics. The filled circles represent the number of boys with influenza in an English boarding school in 1978 (data from the March 4th edition of the *British Medical Journal*). The curves represent solutions from the *SIR* model fitted to the data using least squares. Estimated parameters are $\beta = 1.66$ per day and $1/\gamma = 2.2$ days, giving an R_0 of 3.65.

Information Spreading

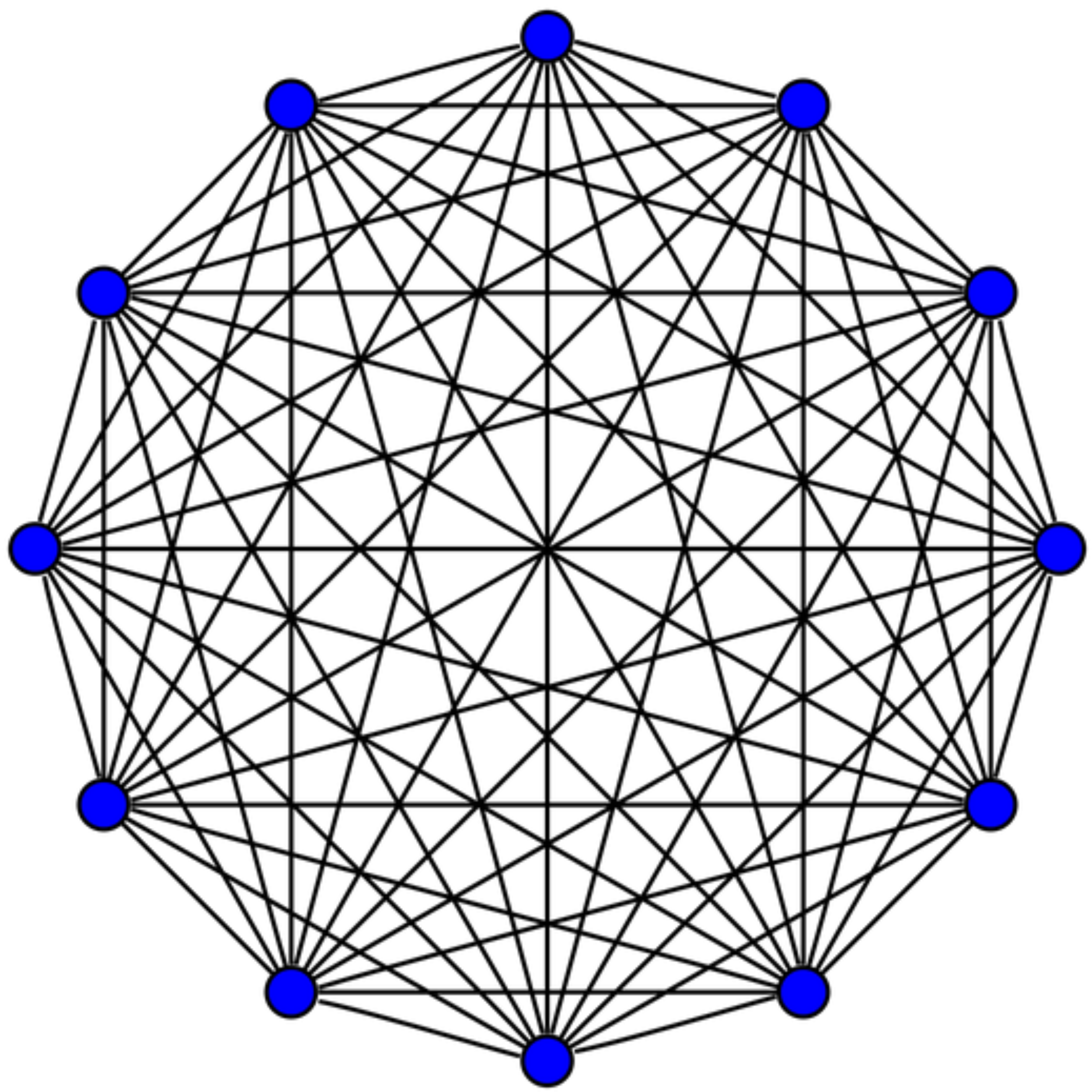
Challenges:

Spreading models + Networks + heteregenous propagation + time delay + stochastic fluctuations + multilayers + …

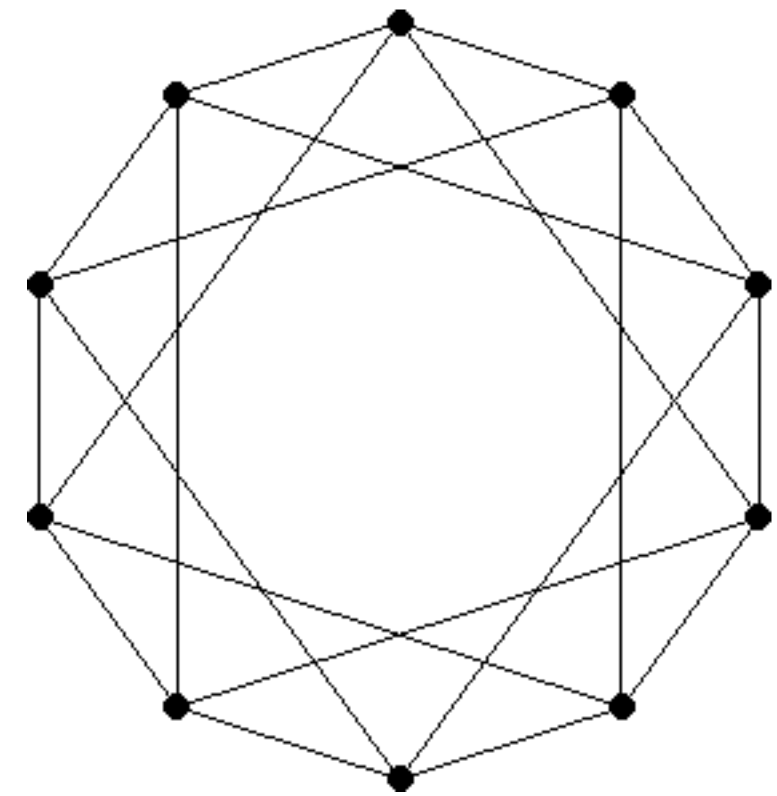
Now:

Spreading models + Networks

Homogeneous networks



Fully connected graph



Each vertex with
k neighbors

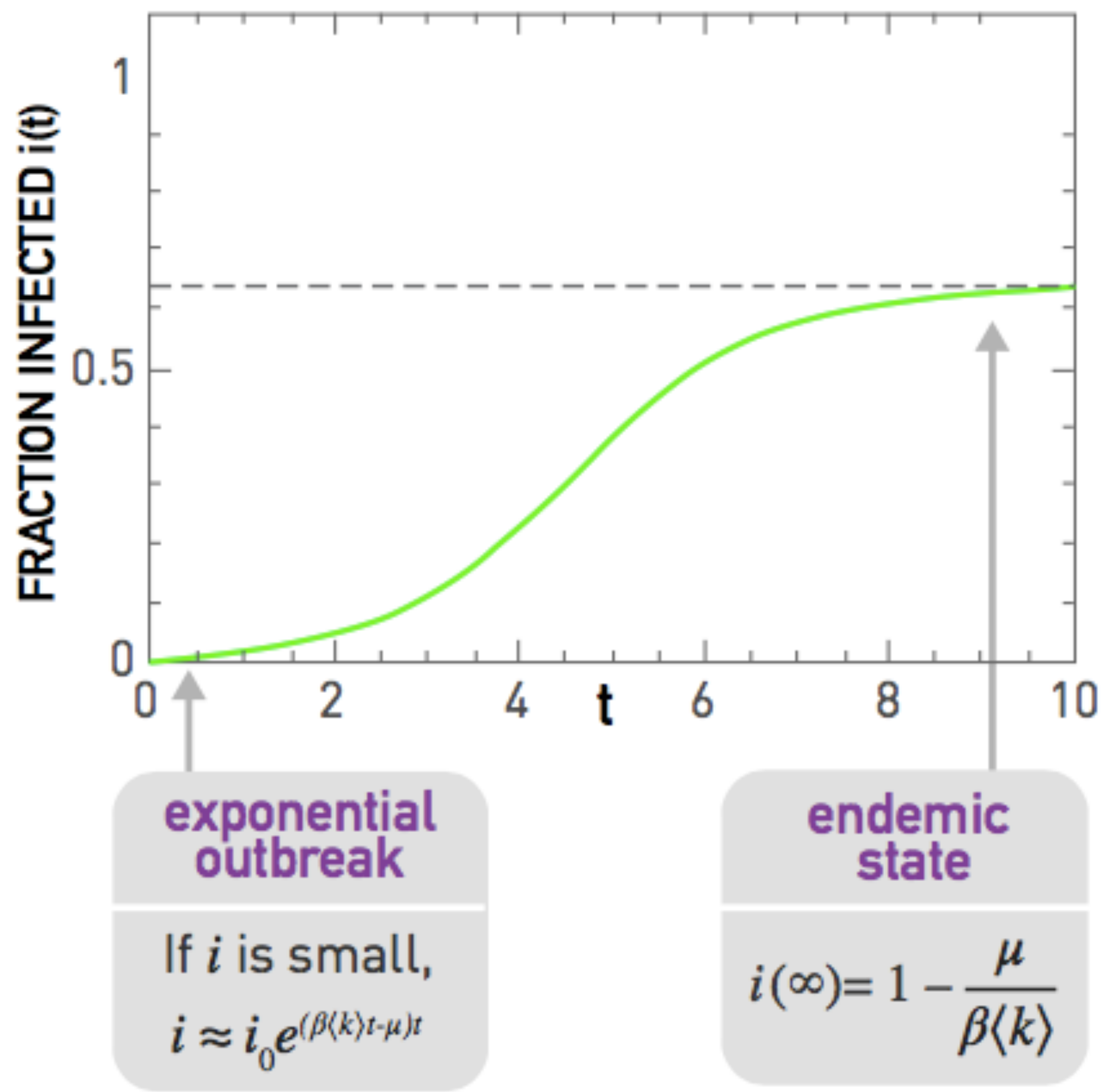
Epidemic Spreading in homogeneous networks

SIS model

$$\frac{di}{dt} = \beta\langle k\rangle i(1-i) - \mu i$$

$$i = \left(1-\frac{\mu}{\beta\langle k\rangle}\right)\frac{Ce^{(\beta\langle k\rangle-\mu)t}}{1+Ce^{(\beta\langle k\rangle-\mu)t}}$$

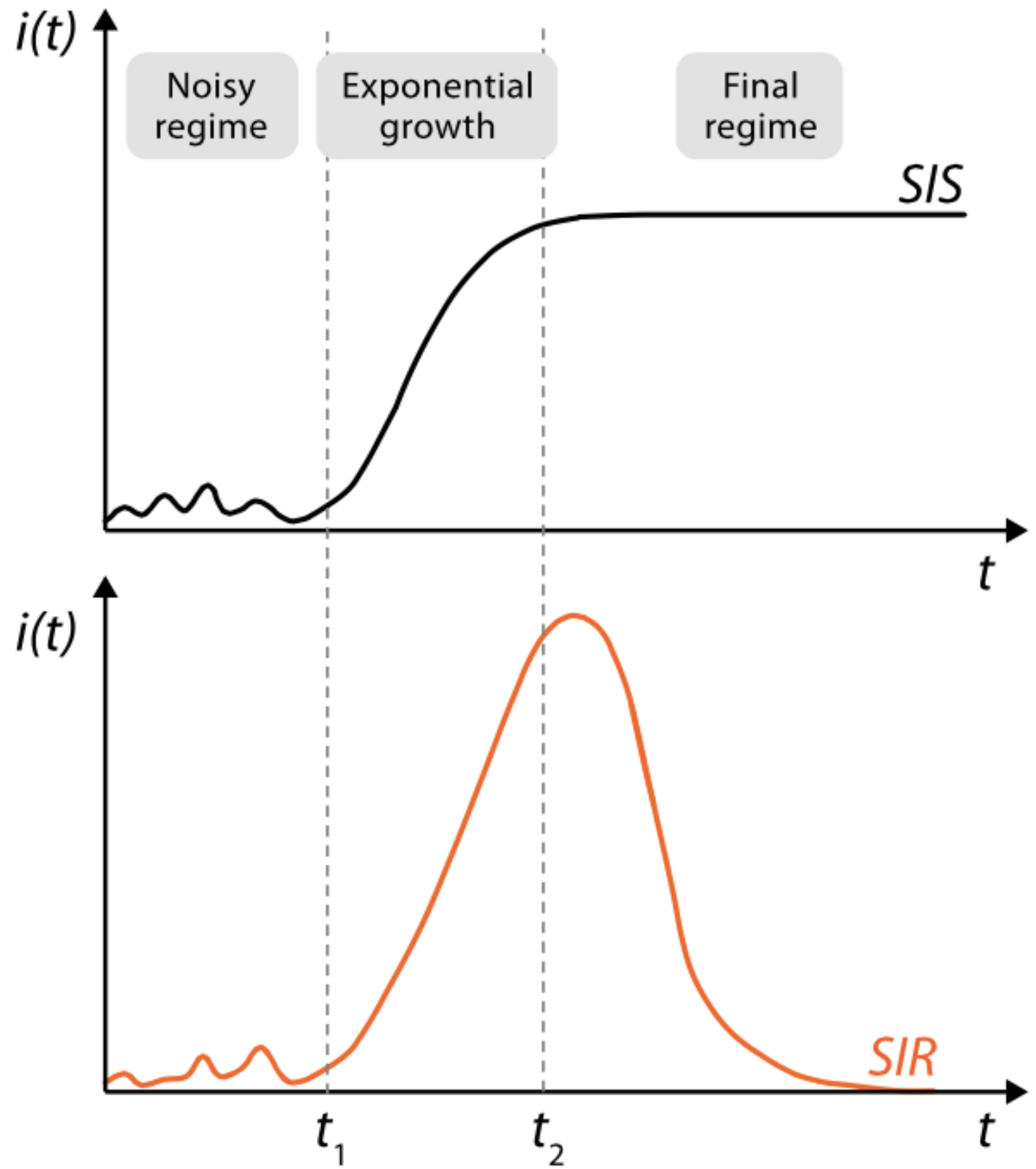
$i_0 = i(t=0)$ gives $C = i_0/(1-i_0-\mu/\beta\langle k\rangle)$



Basic reproductive number $R_0 = \frac{\beta\langle k\rangle}{\mu}$

A. L. Barabási, Network Science, Cambridge, 2015.

SIS X SIR



Pastor-Satorras et al. Reviews of Modern Physics 2015

Epidemic Spreading in homogeneous networks

The SIS model in homogeneous uncorrelated networks at a mean-field level is described by the following rate equation

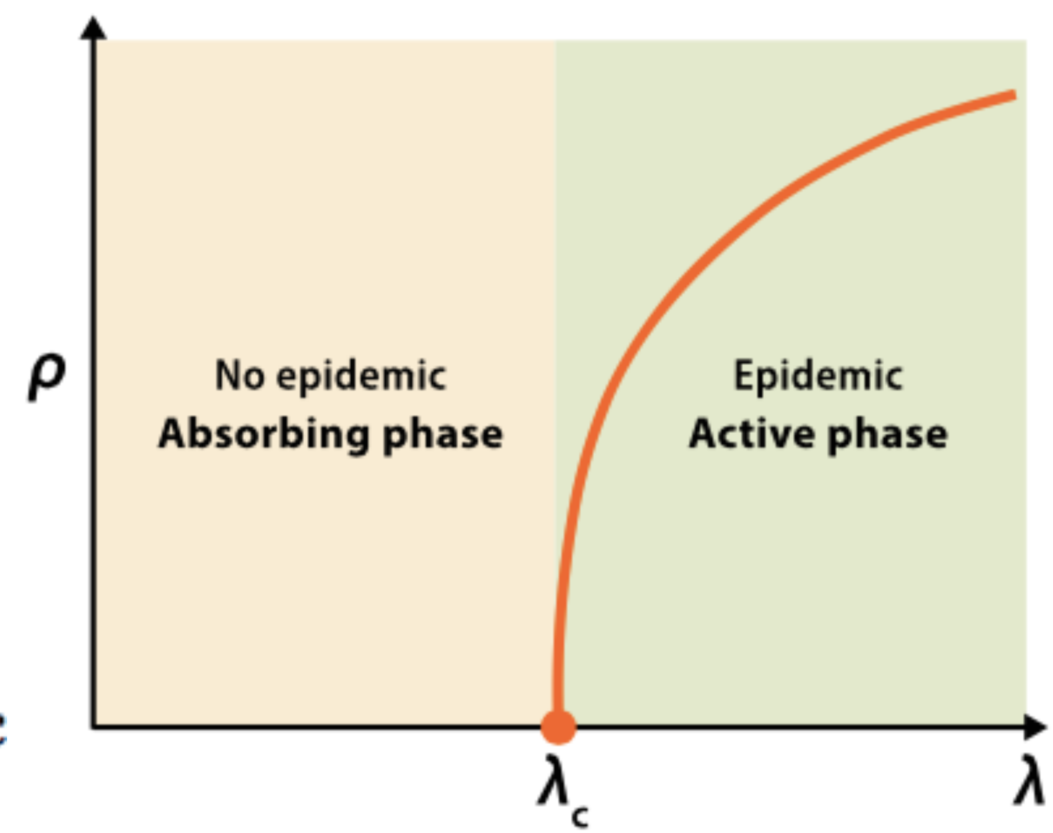
$$\frac{d\rho(t)}{dt} = -\rho(t) + \lambda \langle k \rangle \rho(t) [1 - \rho(t)]$$

it can be proved the existence of an epidemic threshold

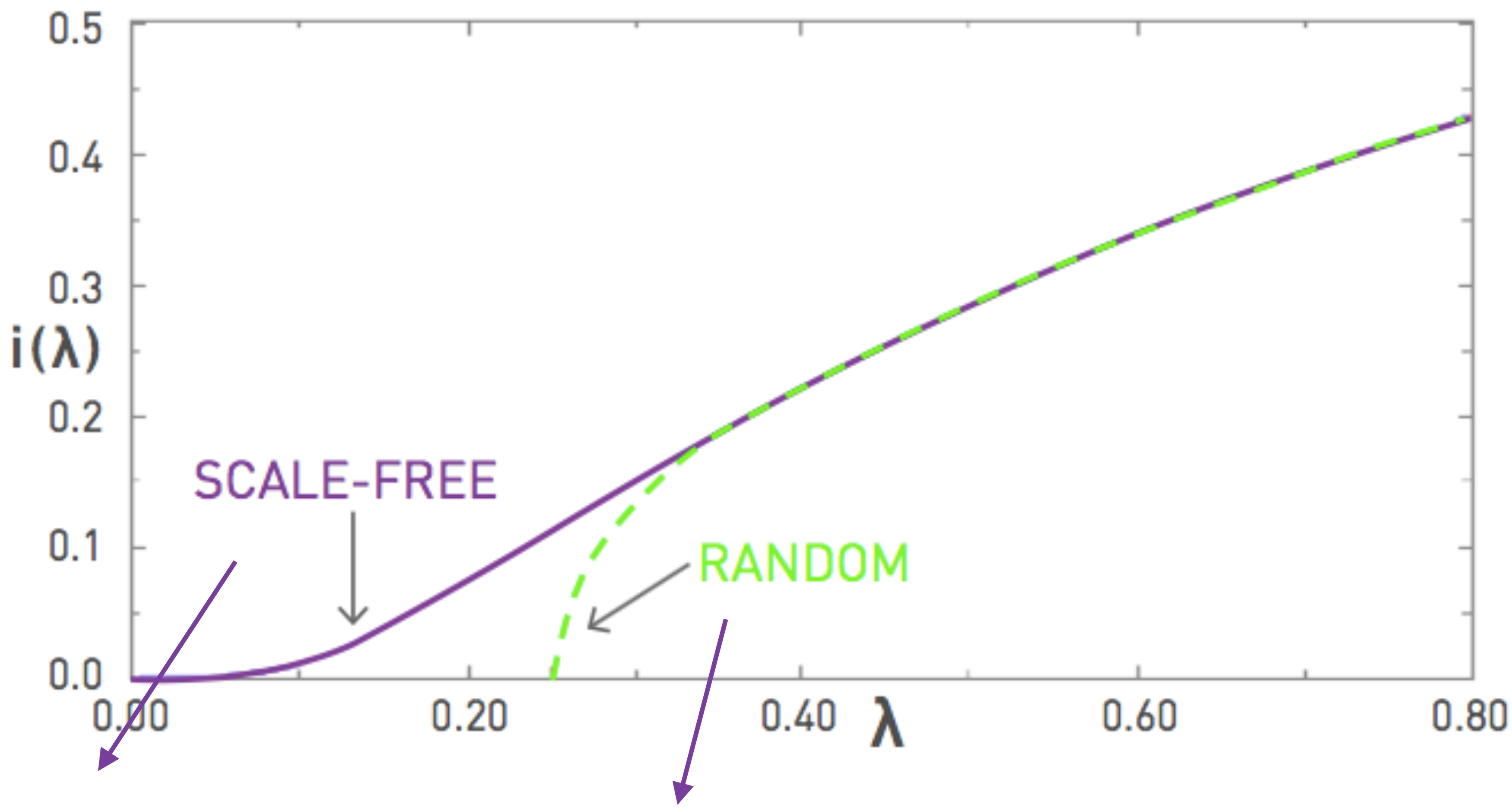
$$\lambda_c = \langle k \rangle^{-1}$$

$$\rho = 0 \text{ if } \lambda < \lambda_c$$

$$\rho \sim (\lambda - \lambda_c) \text{ if } \lambda \geq \lambda_c$$



Epidemic Spreading in heterogeneous networks



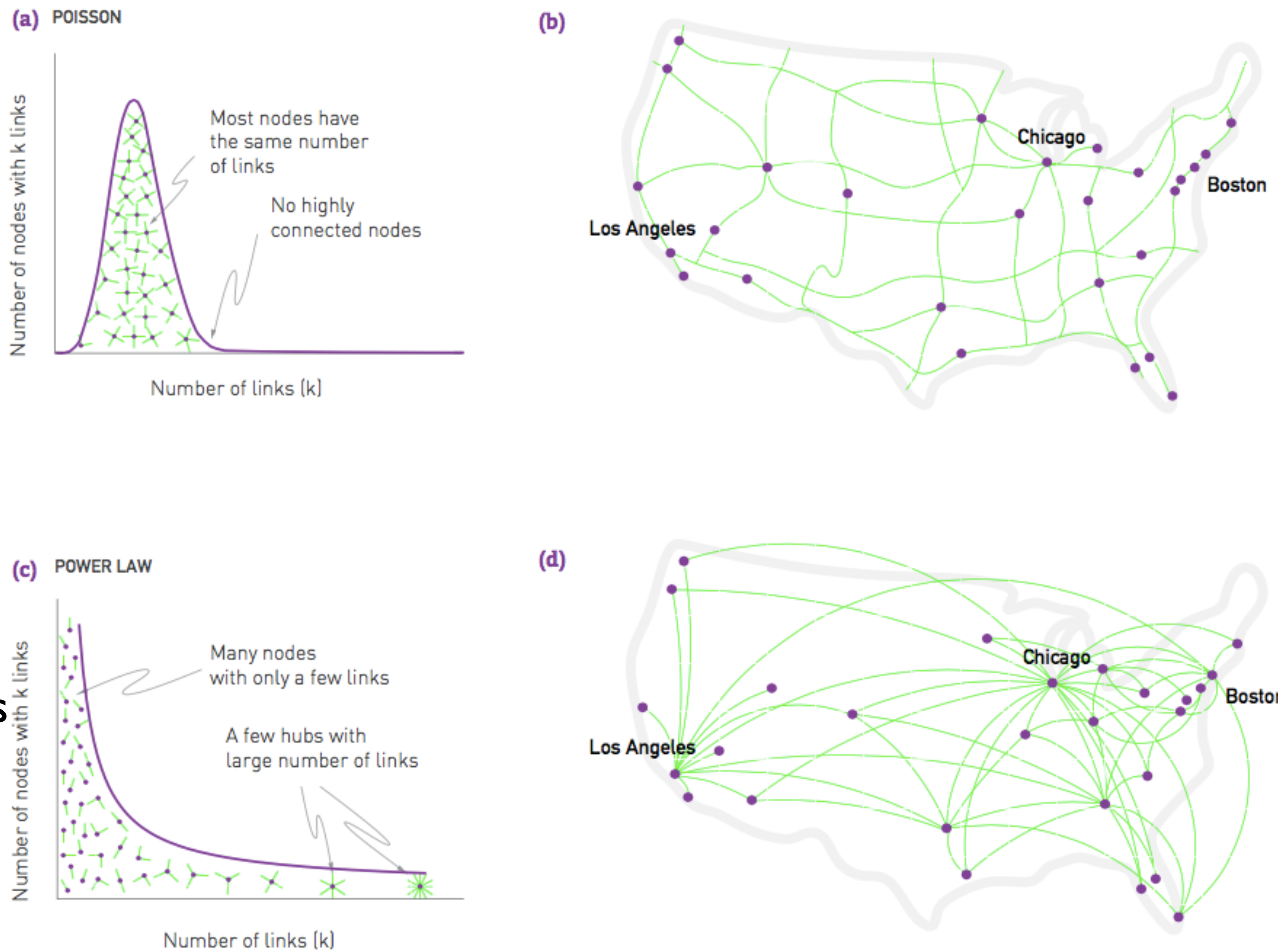
$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle} \sim 0 \qquad \lambda_c = \frac{1}{\langle k \rangle + 1}$$

A. L. Barabási, Network Science, Cambridge, 2015.

Networks

Homogeneous network

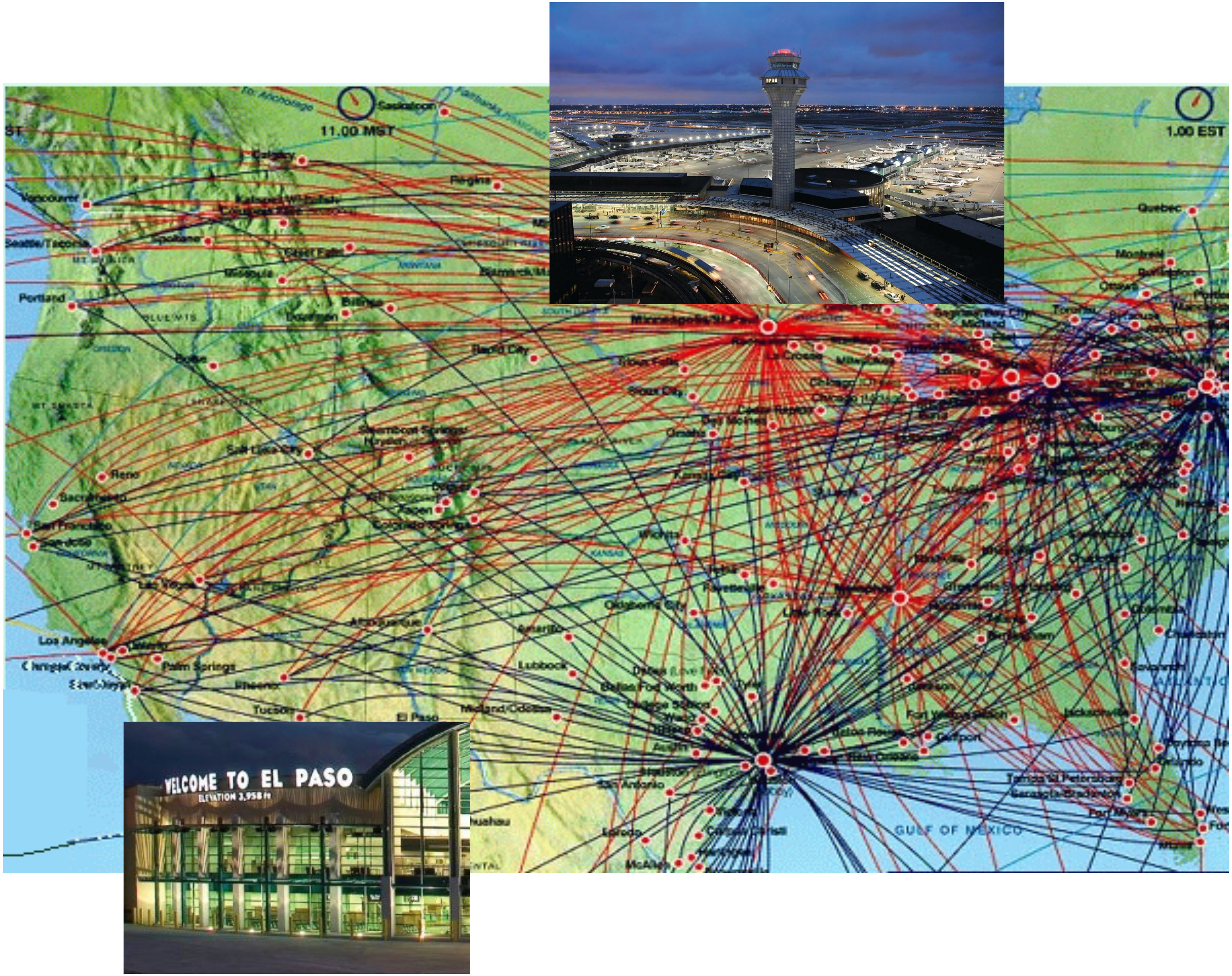
Heterogeneous network



A. L. Barabási, Network Science, Cambridge, 2015.

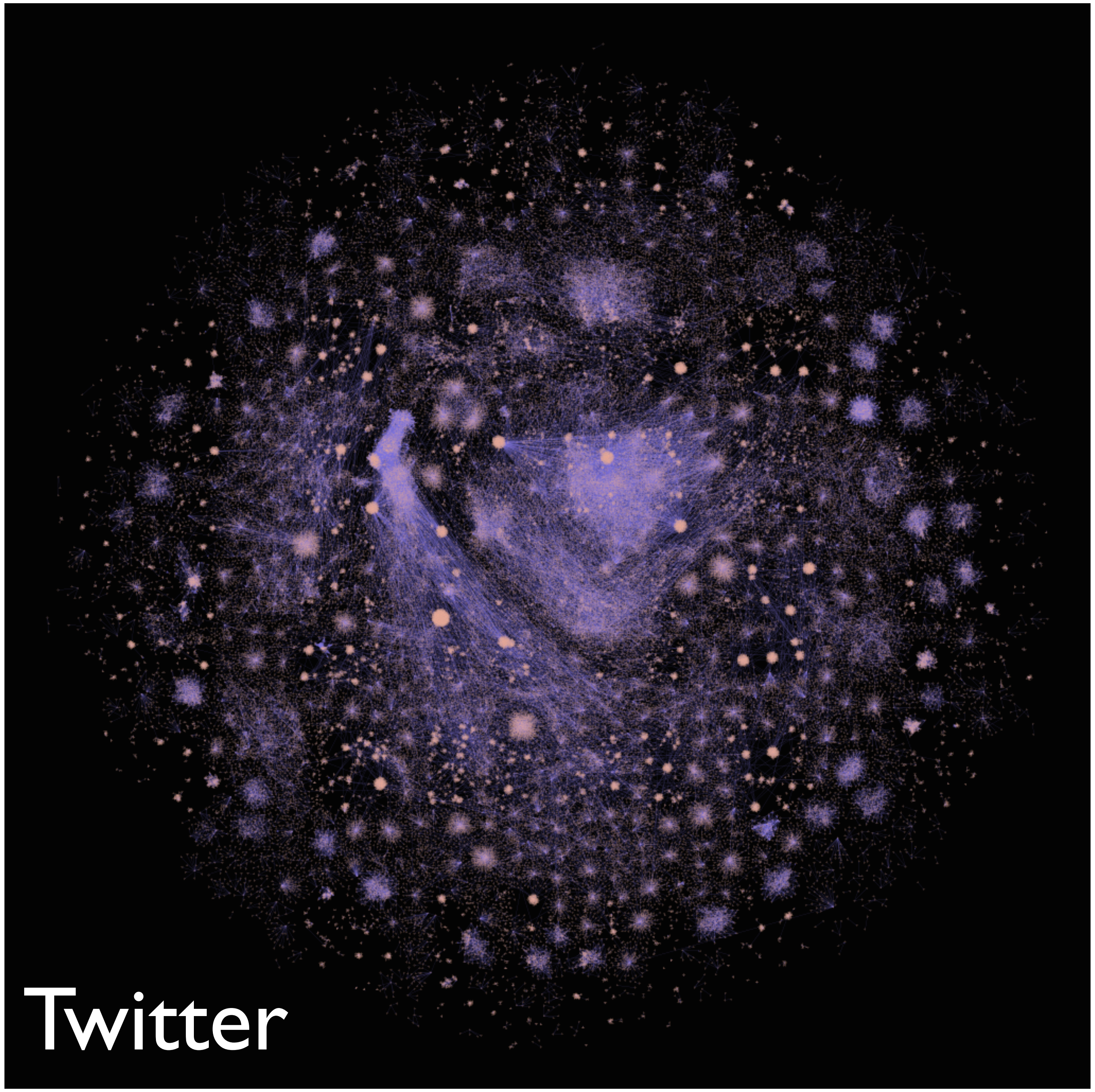
Heterogeneous networks

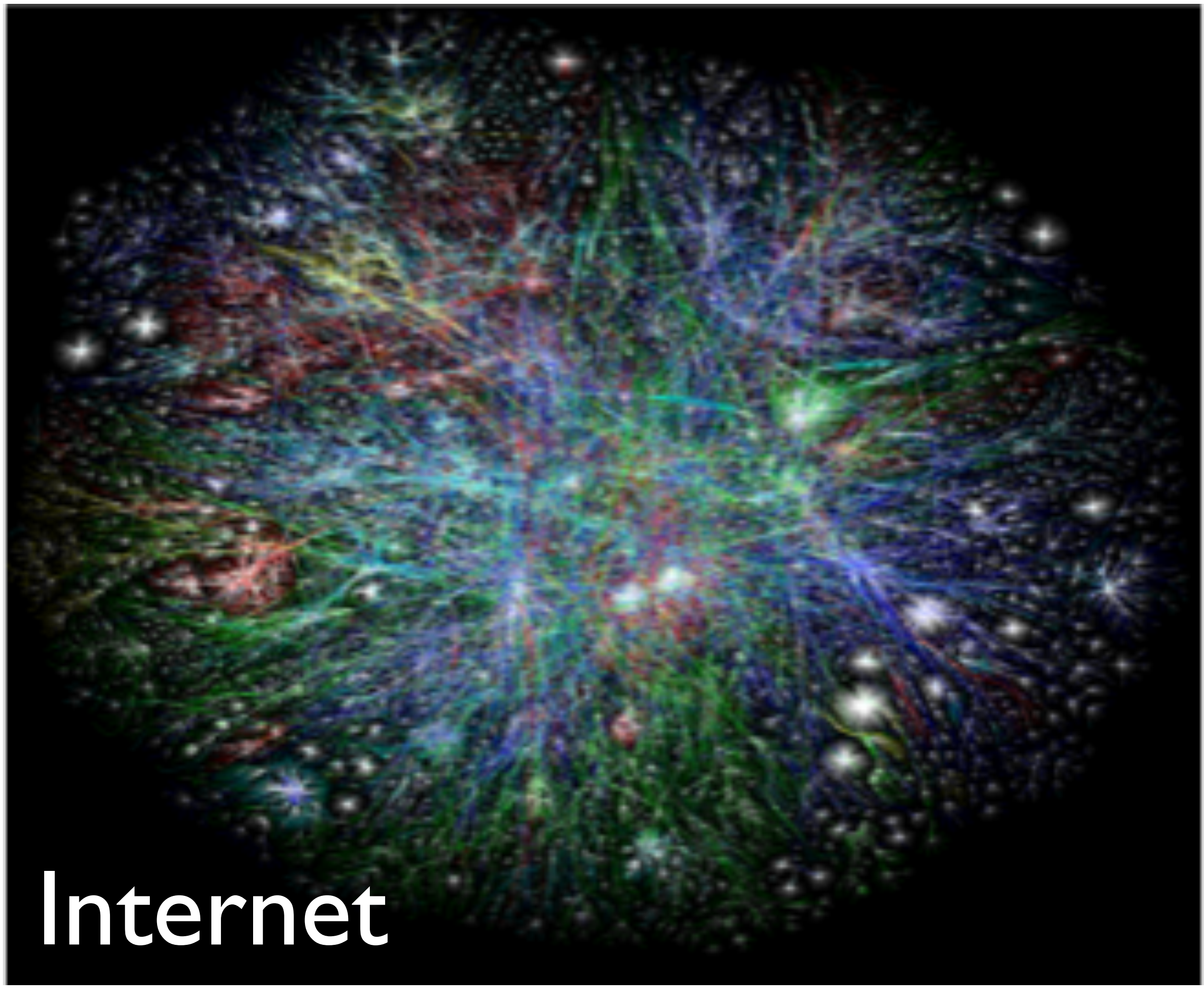
CHICAGO O'HARE INTERNATIONAL AIRPORT



Heterogeneous networks

$$P(k) \approx k^{-\gamma}$$





Deterministic compartmental models

SIR

$$S + I \xrightarrow{\beta} S + S$$
$$S \xrightarrow{\mu} R$$

Maki-Thompson

$$S + I \xrightarrow{\lambda} S + S$$
$$S + S \xrightarrow{\alpha} R + S$$
$$S + R \xrightarrow{\alpha} R + R$$

Daley-Kendal

$$S + I \xrightarrow{\lambda} S + S$$
$$S + S \xrightarrow{\alpha} R + R$$
$$S + R \xrightarrow{\alpha} R + R$$

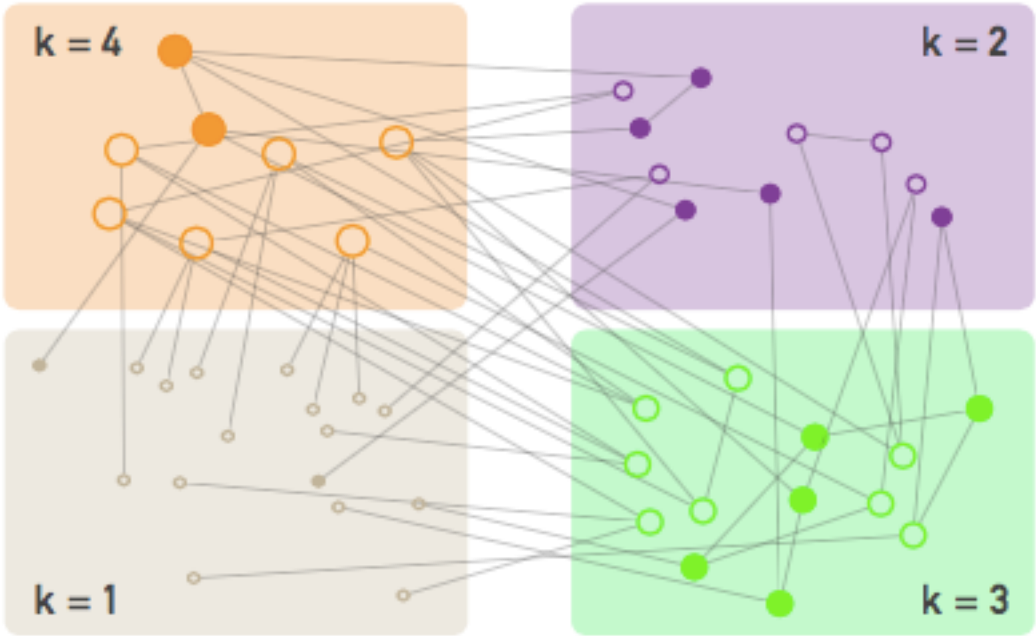
S: Spreader
I: Ignorant
R: Stifler

Rumor spreading in networks

$$\frac{di_k(t)}{dt} = -\lambda k i_k(t) \sum_{k'} \frac{k' P(k') s_{k'}(t)}{\langle k \rangle},$$

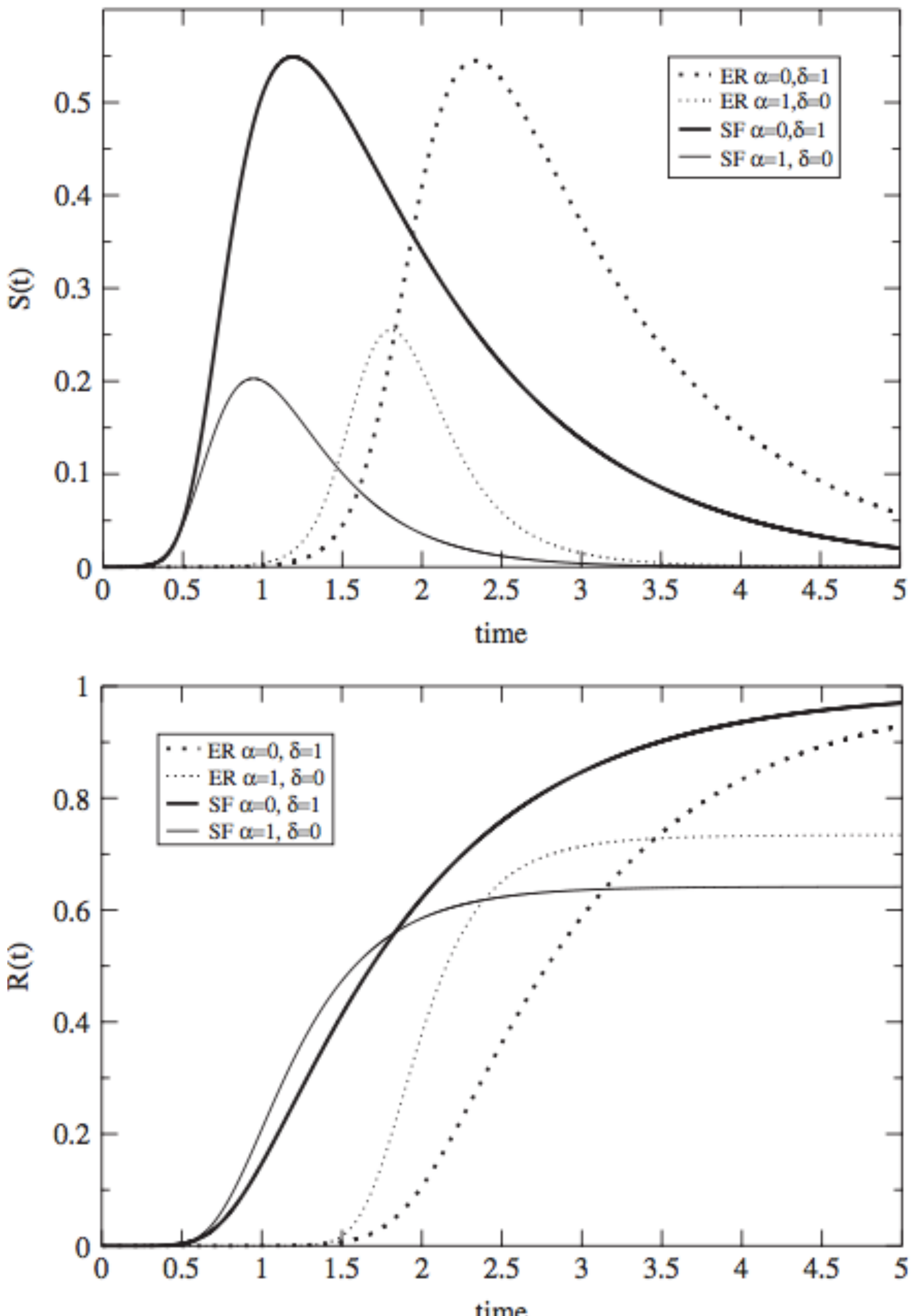
$$\frac{ds_k(t)}{dt} = \lambda k i_k(t) \sum_{k'} \frac{k' P(k') s_{k'}(t)}{\langle k \rangle} - \alpha k s_k(t) \sum_{k'} \frac{k' P(k')[s_{k'}(t) + r_{k'}(t)]}{\langle k \rangle},$$

$$\frac{dr_k(t)}{dt} = \alpha k s_k(t) \sum_{k'} \frac{k' P(k')[s_{k'}(t) + r_{k'}(t)]}{\langle k \rangle},$$



Moreno et al. PRE, 2004

Rumor spreading in networks



S: Spreader

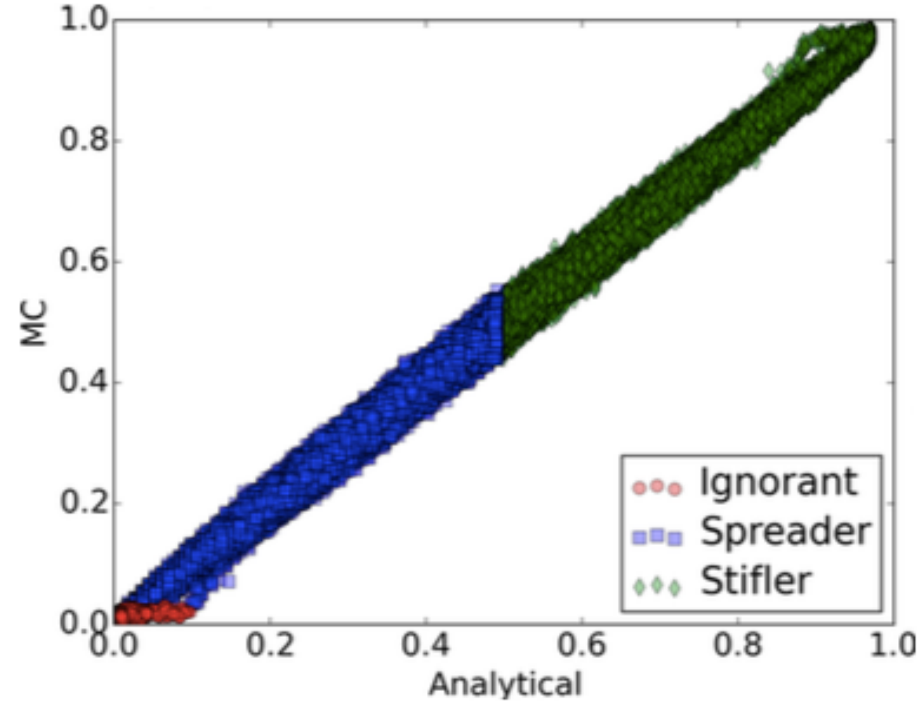
I: Ignorant

R: Stifler

Moreno et al. PRE, 2004

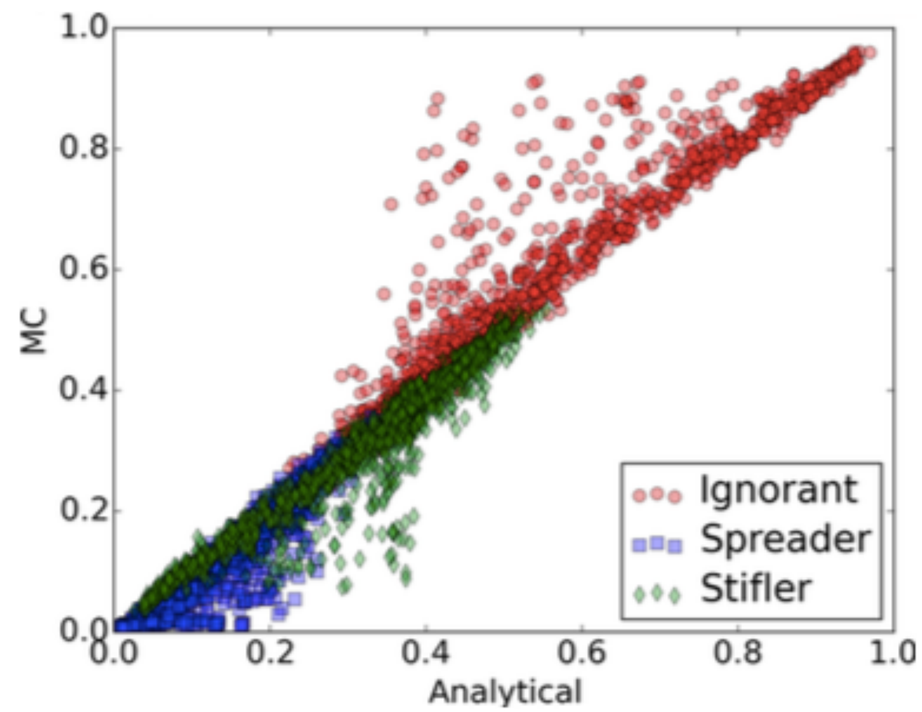
Social Networks

Twitter



(a) RP case on the Twitter network: $\lambda = 1.0$, $\eta = 0.01$, $\delta_1 = \delta_2 = \delta_3 = 0.01$, $\beta = 1.0$ and $\alpha = 1.0$. The probabilities of the final state of the Monte Carlo simulations were estimated by averaging the final state of 10^3 simulations.

E-mail



(b) CP case on the email network: $\lambda = 1.0$, $\eta = 0.01$, $\delta_1 = \delta_2 = \delta_3 = 0.1$, $\beta = 1.0$ and $\alpha = 1.0$. The probabilities of the final state of the Monte Carlo simulations were estimated by averaging the final state of 10^3 simulations.

Social Networks

Twitter: rumor

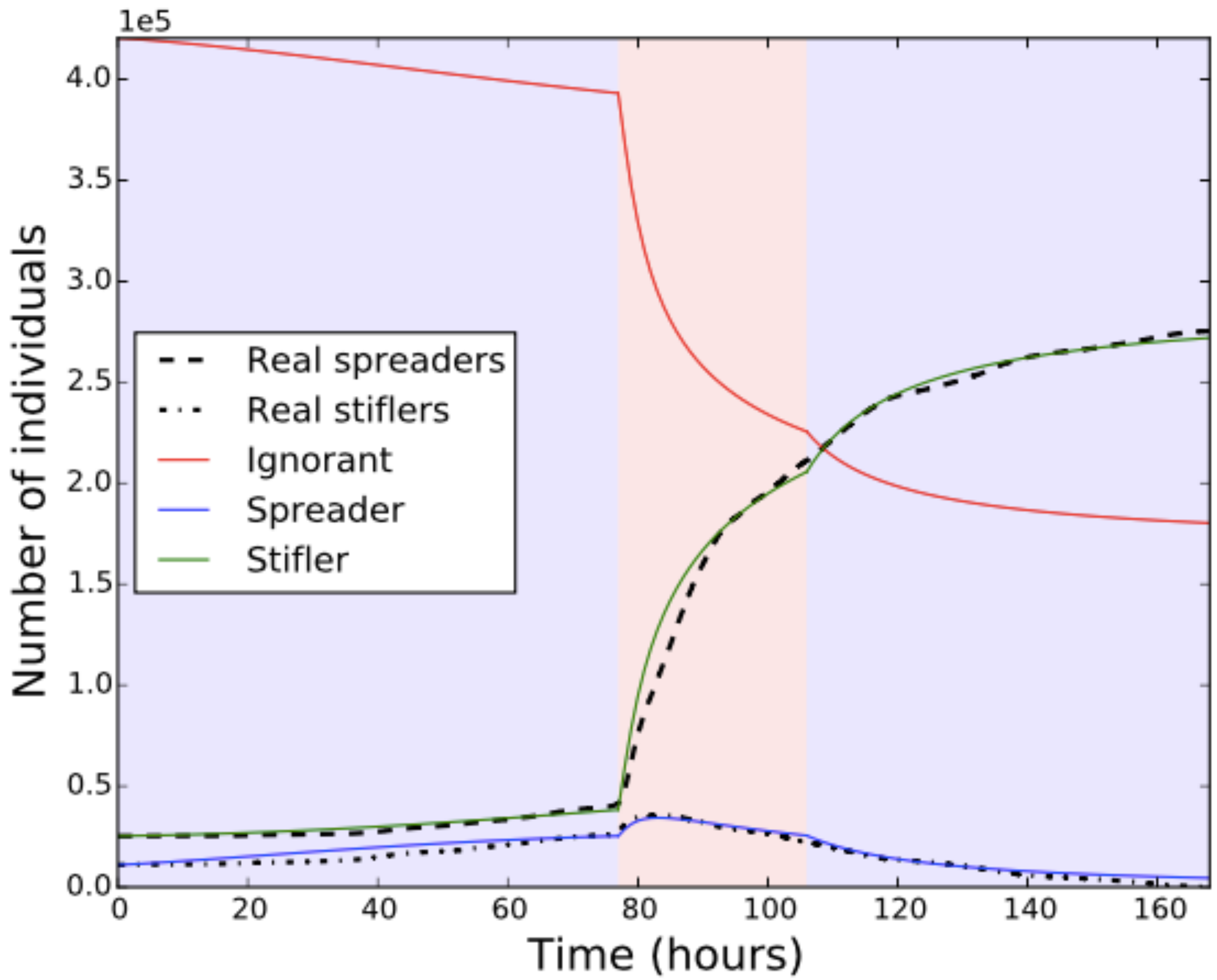
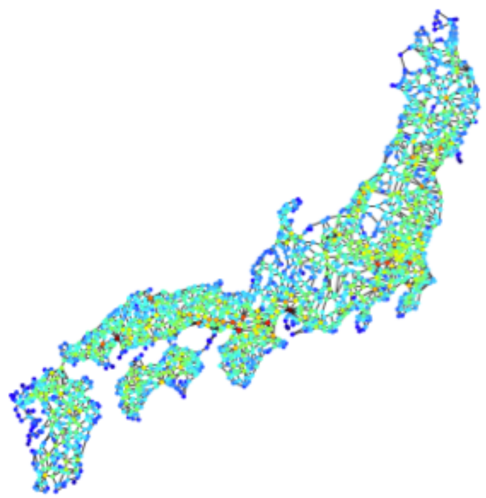


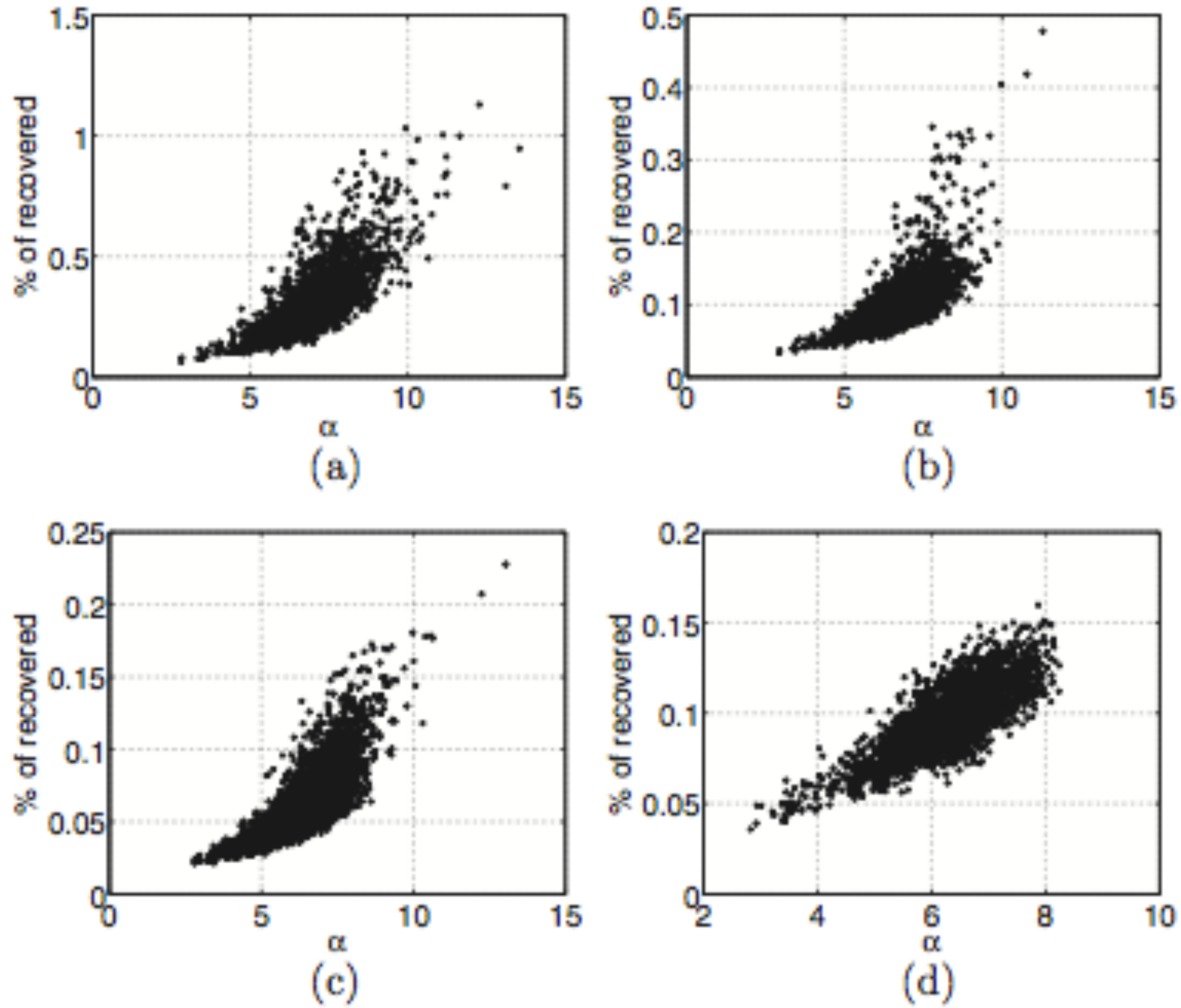
FIG. 16. Time evolution of the Higgs boson rumor spreading, taking into account tweets of the dataset shown in [9]. The dashed lines represent the real data, while the continuous lines are the numeric solution of our model. The background colors discriminates between three time windows, described by different sets of parameters of our model: (i) $0 \leq t \leq 77$, $\lambda = 0.00025$, $\alpha = 0.0002$, $\delta_2 = 0.0001$ and $\eta = 0.85$; (ii) $77 < t \leq 106$, $\lambda = 0.021$, $\alpha = 0.00075$, $\delta_2 = 0.0015$ and $\eta = 0.17$; (iii) $t > 106$, $\lambda = 0.065$, $\alpha = 0.002$, $\delta_2 = 0.002$ and $\eta = 0.01$.

Centrality X Epidemic Spreading

Japan



England



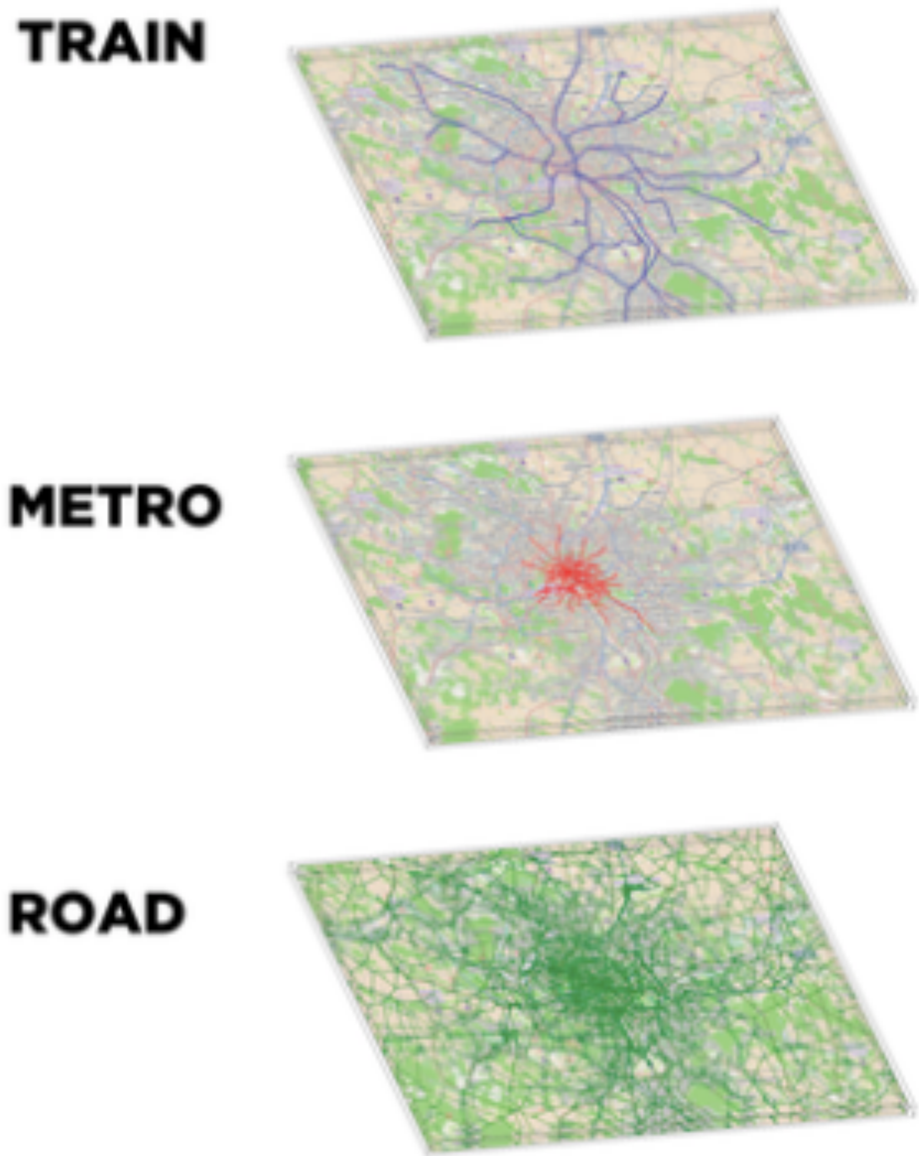
US

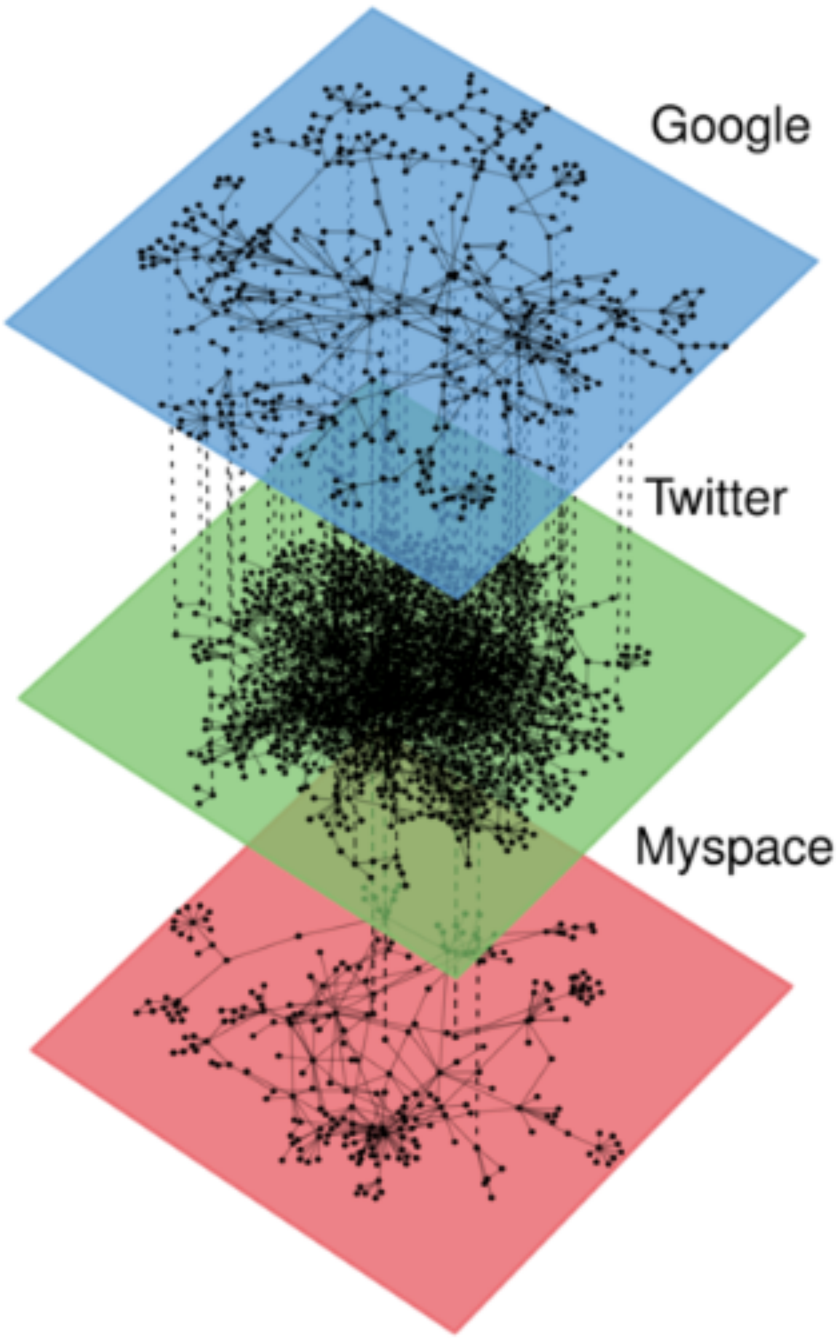
Germany

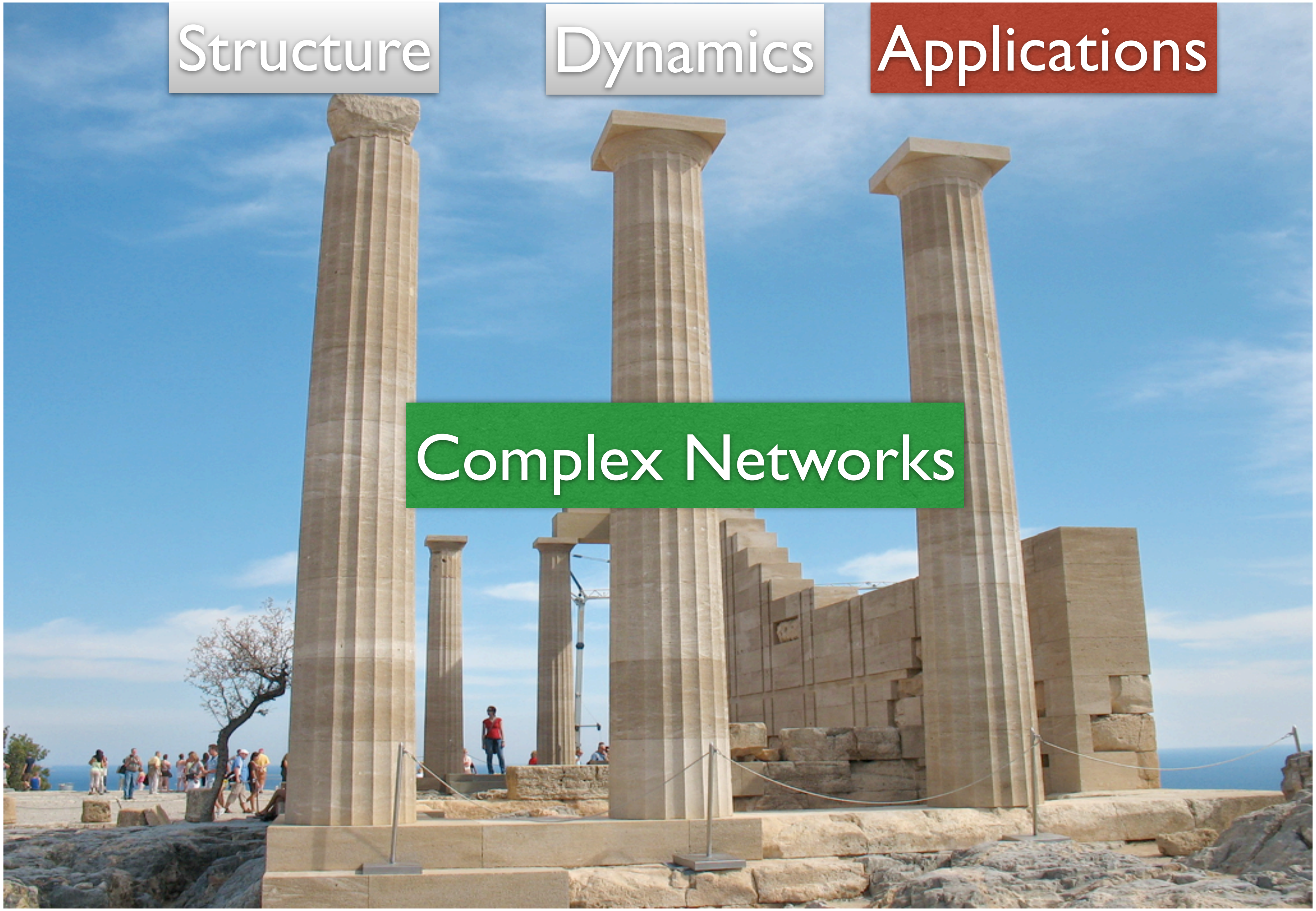
FIG. 7. The percentage of recovered individuals on the SIR epidemic spreading model ($\beta = 0.3, \mu = 1.0$) according to the accessibility measure for the road networks of (a) Japan, (b) England, (c) United States, and (d) Germany.

Arruda et al., PRE, 2014

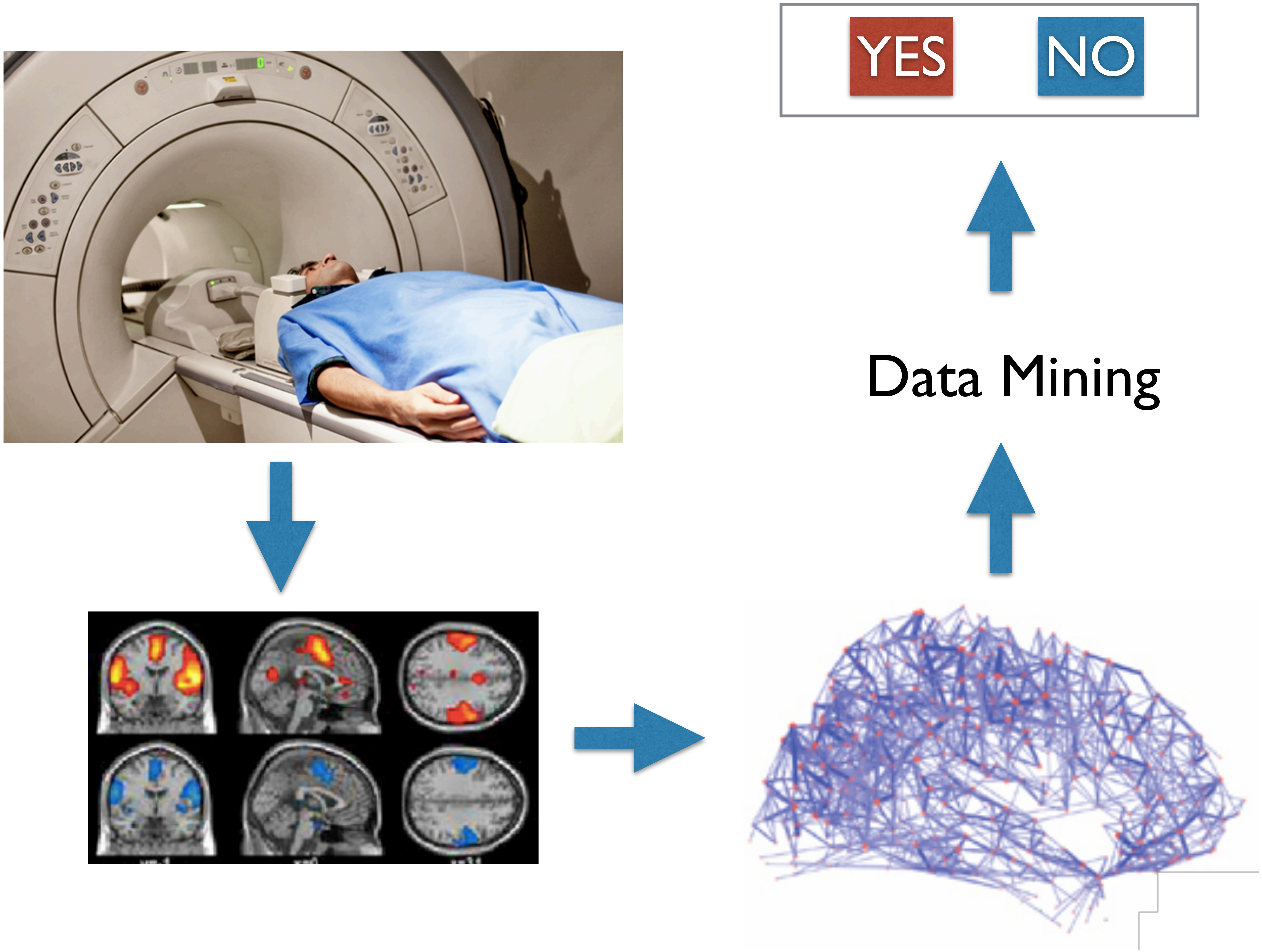
Multilayer networks






Structure
Dynamics
Applications
Complex Networks

The diagnosis of mental disorders

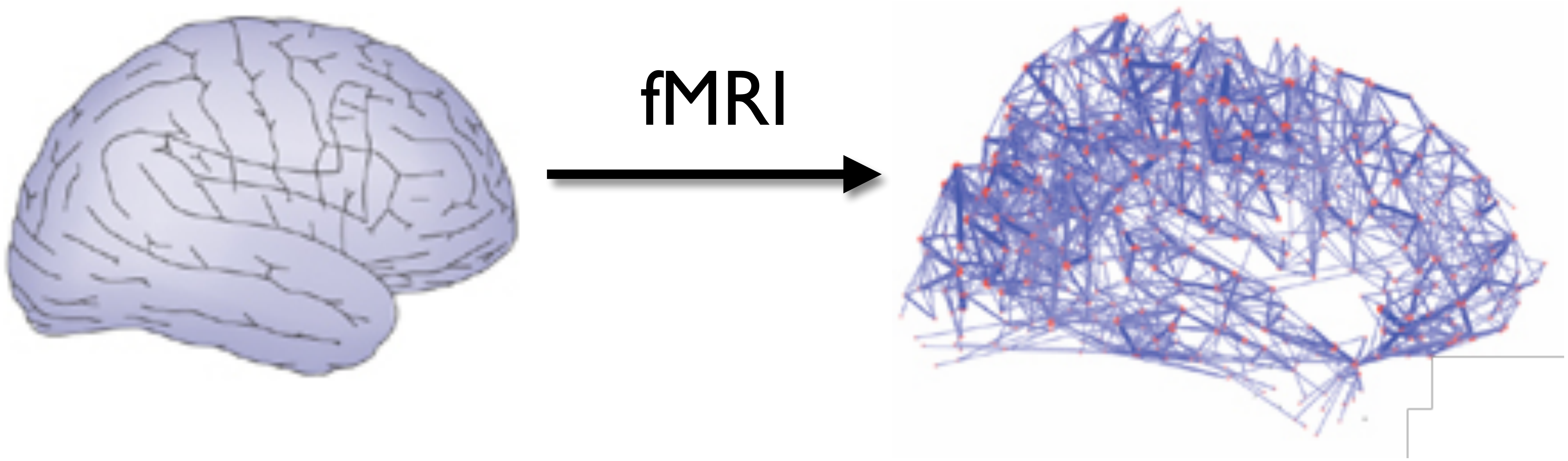


Child-onset schizophrenia

Child-onset schizophrenia
(or pediatric schizophrenia) is a type of mental disorder characterized by degeneration of thinking, motor, and emotional processes in children and adolescents under the age of 18.

Challenge: Early diagnosis.

Child-onset schizophrenia



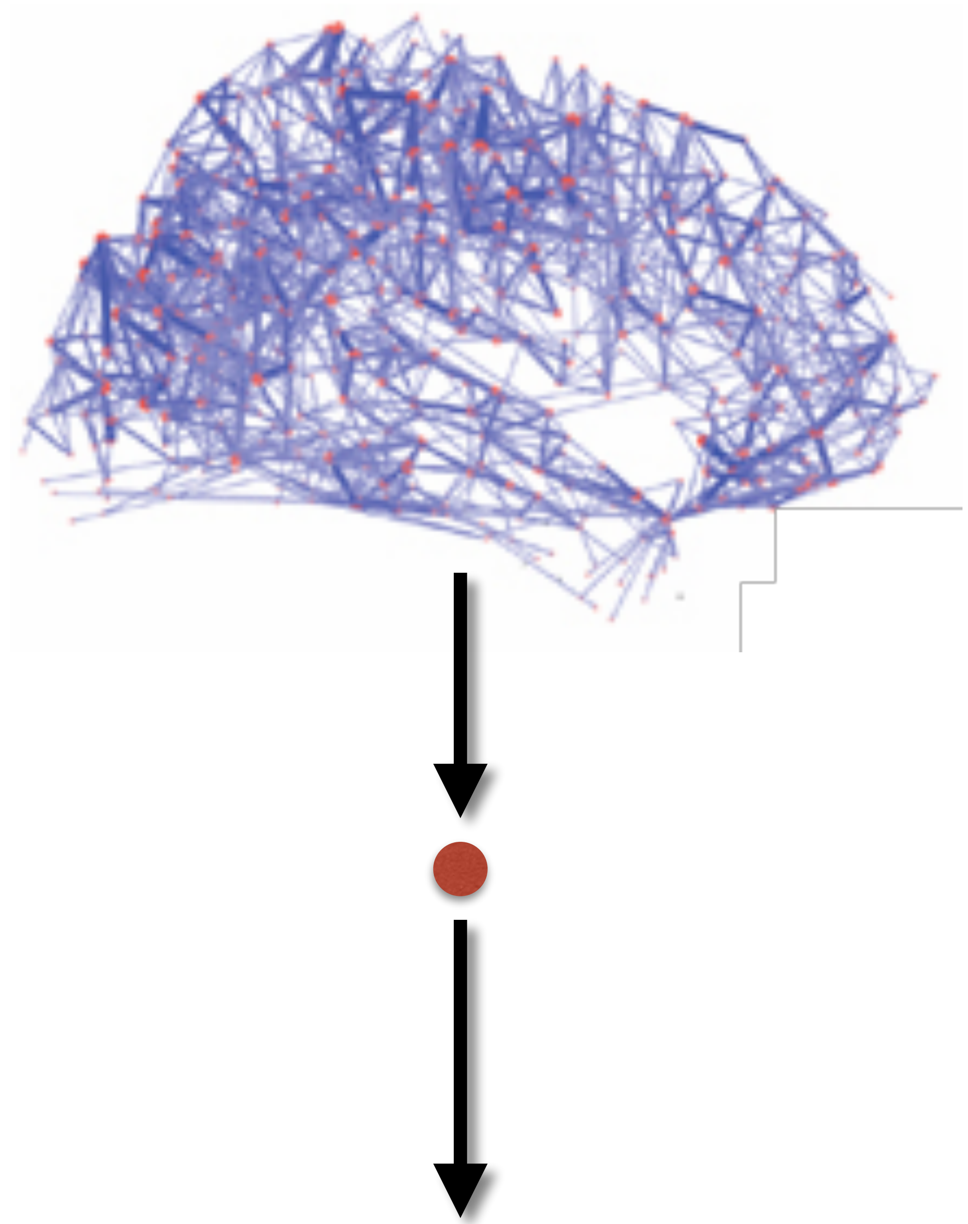
Data: Healthy subjects (n = 20, mean age 19.7 years; 11 male) adolescent participants with childhood-onset schizophrenia (n = 19, mean age 18.7 years; 9 male).

The subjects were scanned using a General Electric Signa MRI scanner operating at 1.5 Tesla.

Only the right hemisphere (140 regions).

Vértes et al. PNAS, 2011

Child-onset schizophrenia



Vi = [M1, M2, M3, …, M54]

54 measures calculated for each node.

Child-onset schizophrenia

Table 1: Feature ranking of network measures calculated by using symmetrical uncertainty (U) and chi-squared test ($\mathcal{X}^2$). The features are ordered according to the symmetrical uncertainty.

$U(C,A)$	$\mathcal{X}^2$	Feature
0.326	15.55	Variance of the closeness centrality
0.289	10.13	First moment of K-core
0.263	12.88	Modularity
0.258	12.74	Variance of the accessibility

Table 2: Percentage of correct classification of networks obtained from healthy and schizophrenic subjects considering 4 or 54 measures. PC is the positive class, $H.$ indicates the healthy class and $S.$, schizophrenic subjects.

	Naive Bayes		Bayesian network		C4.5 Decision tree	
	54 meas.	4 meas.	54 meas.	4 meas.	54 meas.	4 meas.
Accuracy	0.74	0.76	0.71	0.78	0.45	0.71
Precision (PC: H.)	0.68	0.73	0.70	0.76	0.46	0.68
Specificity: Recall (PC: H.)	0.90	0.84	0.74	0.84	0.58	0.79
F-Measure (PC: H.)	0.77	0.78	0.72	0.80	0.51	0.73
Precision (PC: S.)	0.85	0.81	0.72	0.82	0.43	0.75
Sensitivity: Recall (PC: S.)	0.58	0.68	0.68	0.74	0.32	0.63
F-Measure (PC: S.)	0.69	0.74	0.70	0.78	0.36	0.69

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Structure and dynamics of functional networks in child-onset schizophrenia

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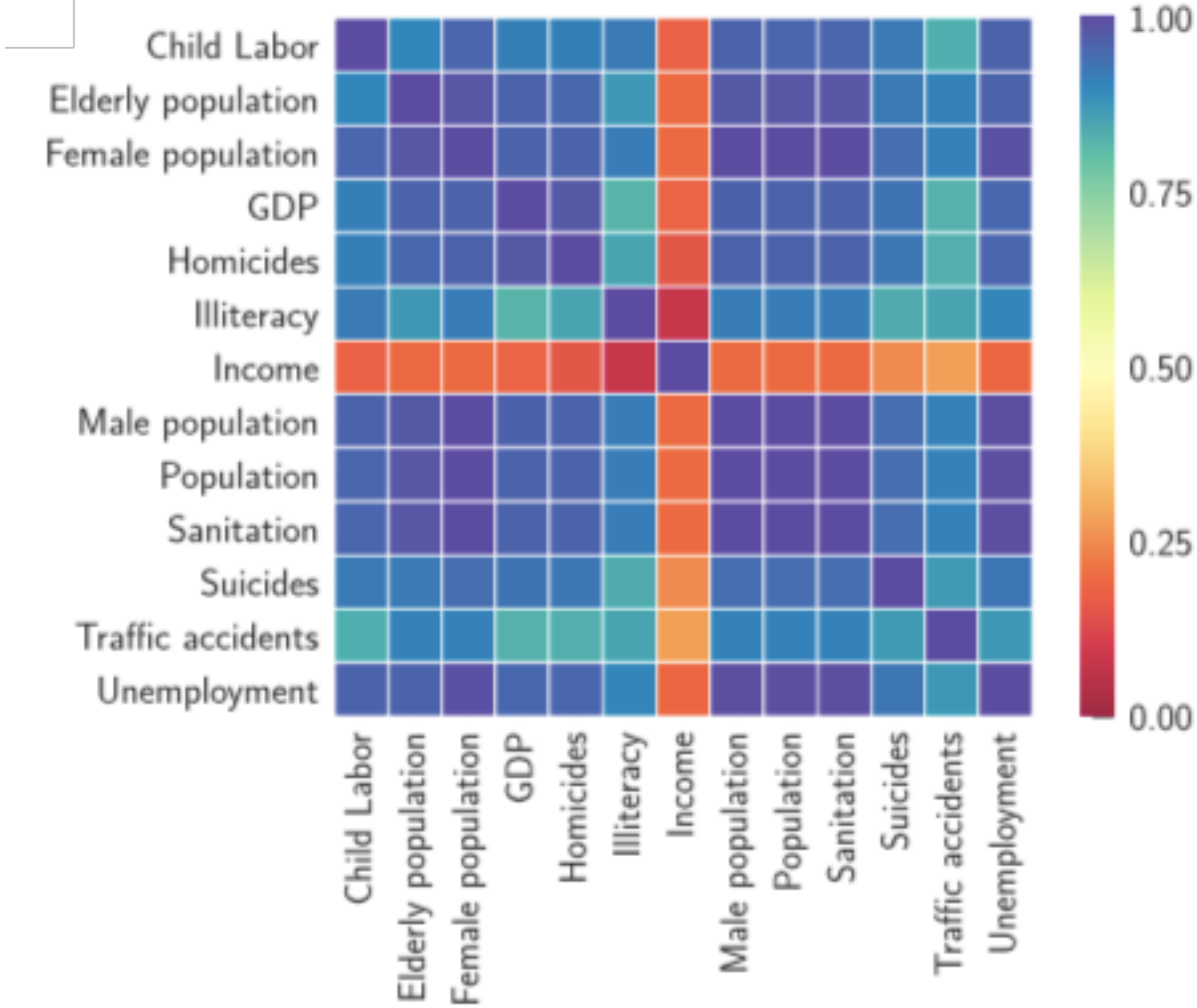
HIGHLIGHTS

- Cortical networks in healthy and child-onset schizophrenic subjects differ significantly in only a small number of large-scale network properties.
- Classification of networks of healthy and schizophrenic subjects yields sensitivity of 90% and specificity of 74%.
- Our analysis allows reliable automatic diagnostics in patients with child-onset schizophrenia based on cortical network properties and data mining methods.

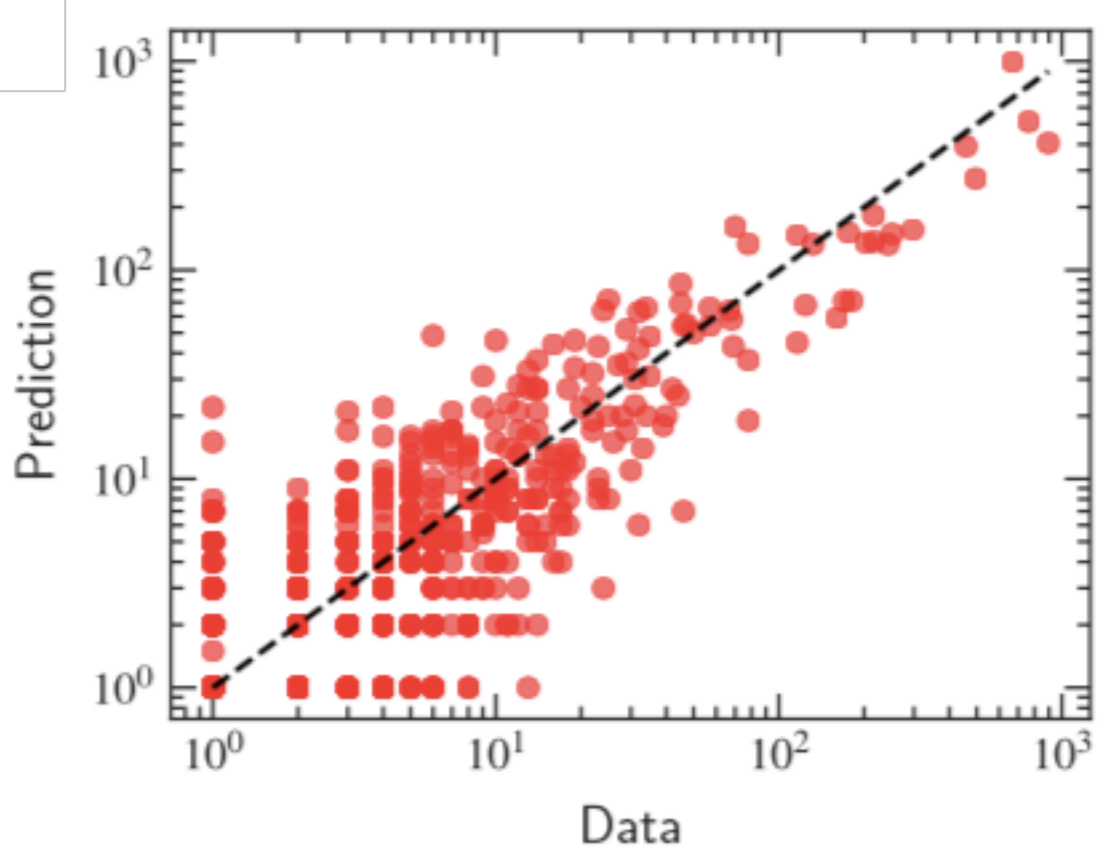
Arruda et al. Clinical Neurophysiology, 2013

Crime Prediction



$$H(t+\Delta t) = \frac{1}{B}\sum_{b=1}^{B} T(H(t), X_1(t), \ldots, X_n(t); \Theta_b),$$

Random forest



Alves, Ribeiro and Rodrigues, https://arxiv.org/abs/1712.03834

Crime Prediction

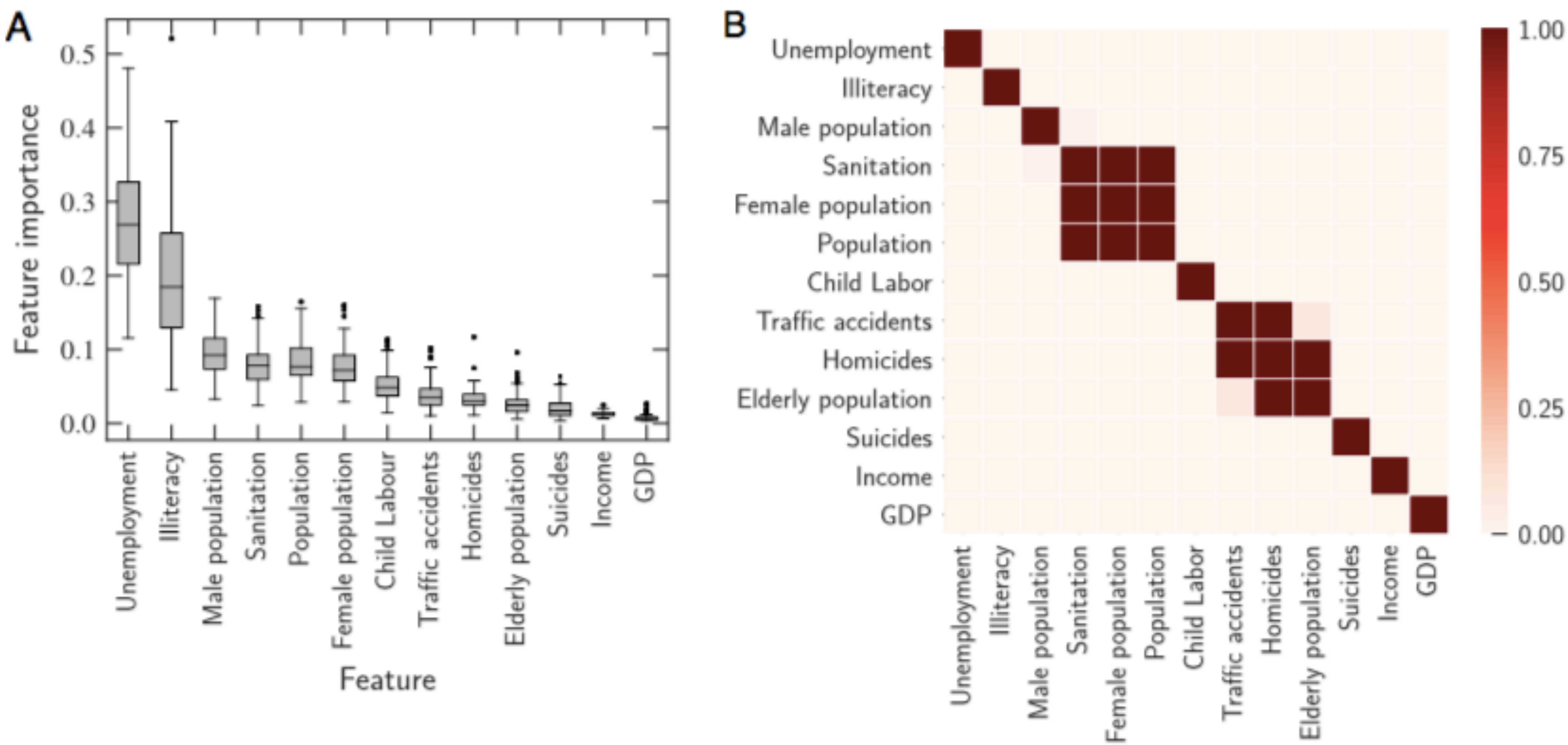
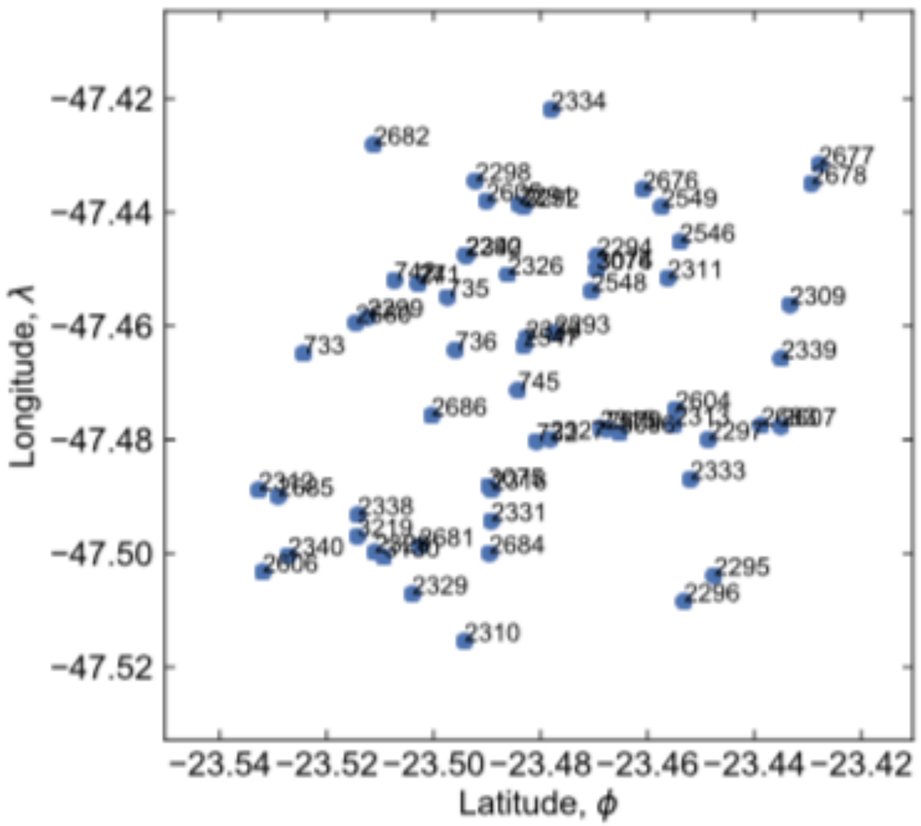


Figure 4: The important urban metrics for predicting the crime of homicide. A) The box-plot shows the rank the importance of urban metrics by their medians over different random sample splits. The black lines dividing the boxes represent the median, boxes represent the interquartile range, upper and lower whiskers bars are the most extreme non-outlier data points, and dots are the outliers. B) The matrix plot shows the $p-$values of $t-$Student test comparing whether the positions in the rank are different. We identify four groups of features that are equally important in the rank (squares on the diagonal matrix).

Alves, Ribeiro and Rodrigues, https://arxiv.org/abs/1712.03834

Traffic forecasting model

Position of the cameras



+

City map (Network)

$$F_i(t + \Delta t) = \alpha_i F_i(t) + \sum_{i=1}^{n} \beta_i F_j(t) + \epsilon$$

Number of cars

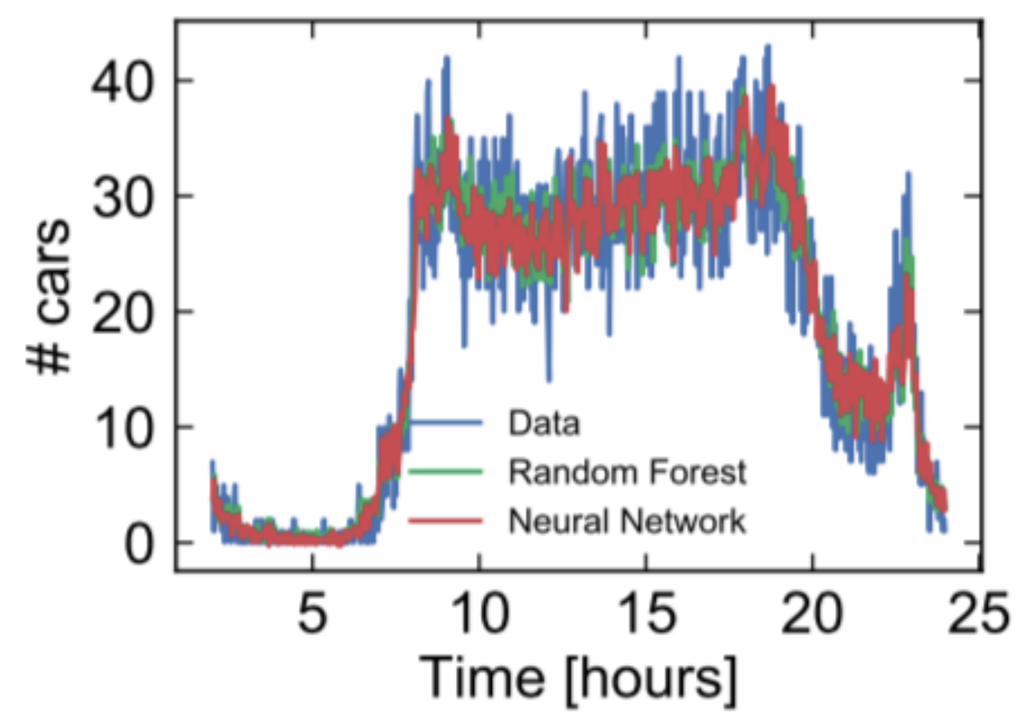
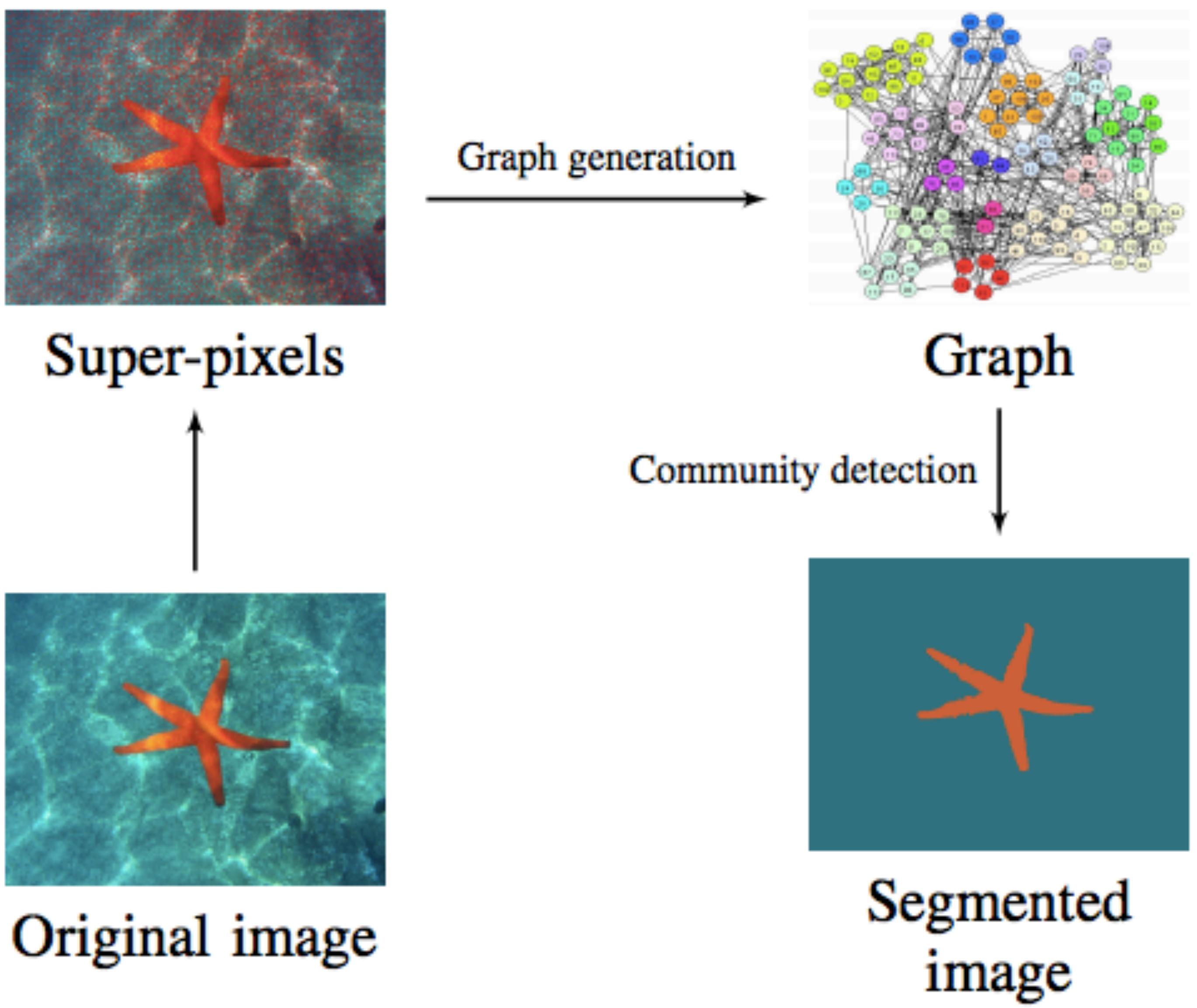
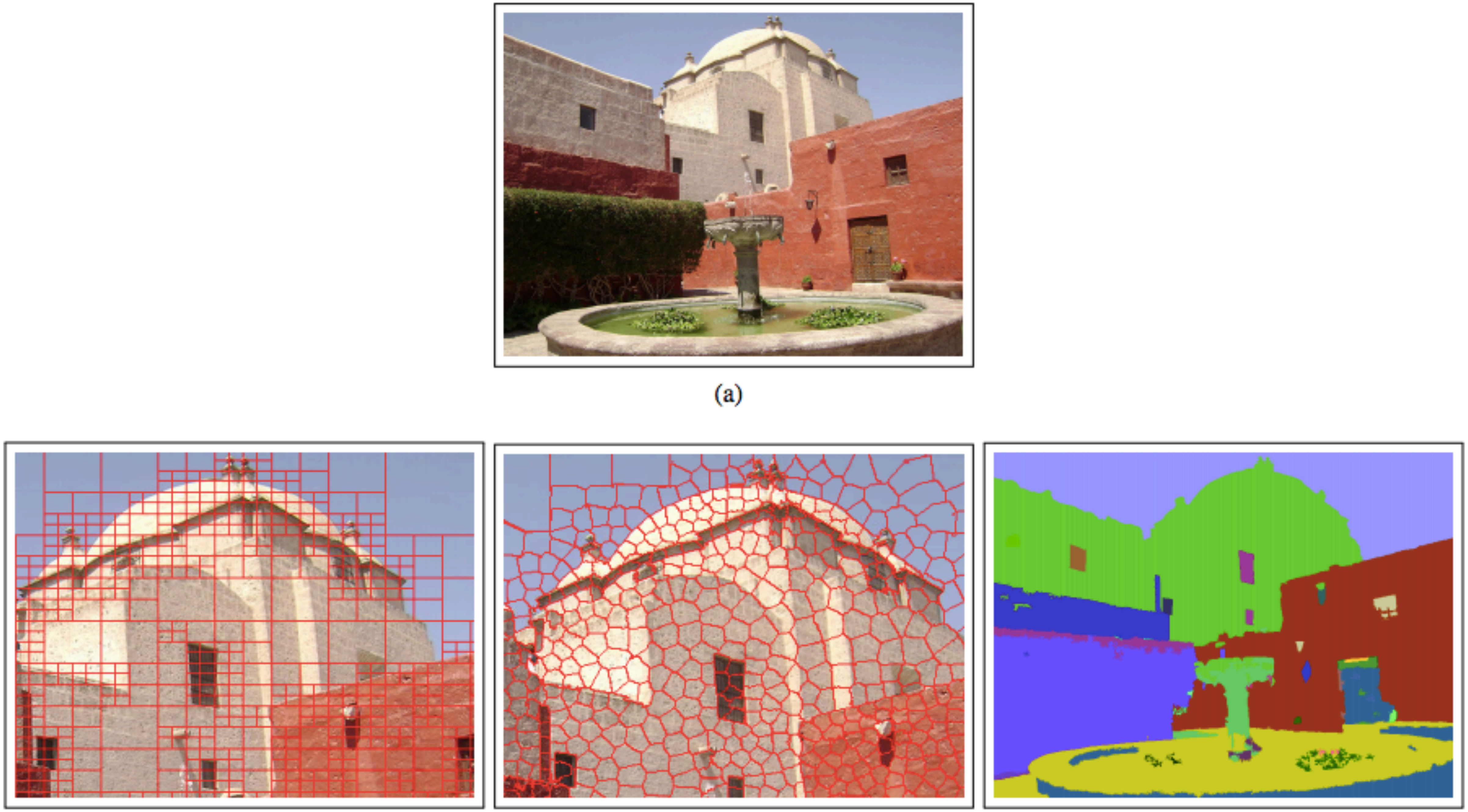


Image segmentation



Linhares et al., IET Image Processing, Volume 11, Issue 12 (2017).

Image segmentation

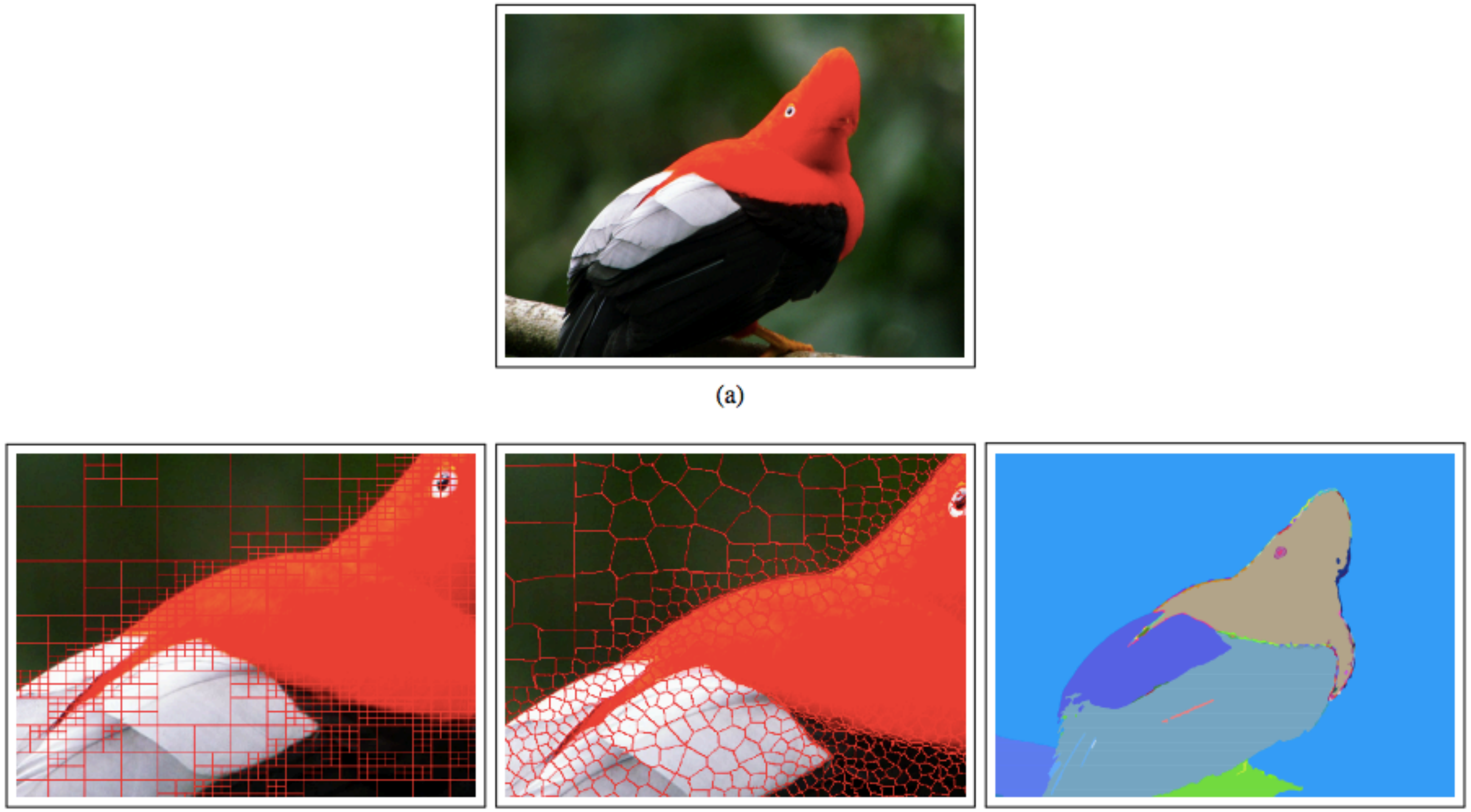


(a)

(b) (c) (d)

Linhares et al., IET Image Processing, Volume 11, Issue 12 (2017).

Image segmentation



(a) (b) (c) (d)

Method	Mean	Standard Deviation
Felzenszwalb	0.48	0.52
Arbelaez	0.67	0.16
Community Detection	0.74	0.13

Table 2 Segmentation quality Mean and standard deviation computed for the Berkeley database.

Other works

- Epidemic spreading with awareness
- Synchronization of moving agents
- Climate Networks
- World Trade Networks
- Diagnosis of mental disorders (autism,bipolar disorder)
- Ecological networks
- …

Quer saber mais?

- SME 130 Redes Complexas

Thank you!

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