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Local mountain pass for a class of elliptic systems

by

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Our aim is to show a result involving the existence and concentration of positive solutions for the following class of elliptic systems

$$(S) \quad \begin{cases} -\epsilon^2 \Delta u + W(x)u = Q_u(u, v) & \text{in } \mathbb{R}^N \\ -\epsilon^2 \Delta v + V(x)v = Q_v(u, v) & \text{in } \mathbb{R}^N \\ u(x), v(x) \rightarrow 0, & \text{as } |x| \rightarrow \infty \\ u, v > 0 & \text{in } \mathbb{R}^N \end{cases}$$

where V, W are nonnegative Hölder continuous functions and Q is a p -homogeneous function with $2 < p < 2N/(N-2)$ and $N \geq 3$.

The main result completes a result due Del Pino & Felmer [1] for the problem

$$\begin{cases} -\epsilon^2 \Delta u + V(x)u = |u|^{p-2}u, & \text{in } \Omega. \\ u = 0, & \text{on } \partial\Omega \end{cases}$$

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Asymptotic behavior at the boundary of solutions of reaction-diffusion equations with nonlinear boundary conditions

by

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We consider a reaction diffusion equation with nonlinear boundary condition of the type

$$\begin{cases} u_t - \Delta u = f(x, u), & \Omega \\ \frac{\partial u}{\partial n} = g(x, u), & \partial\Omega \end{cases}$$

in a bounded smooth domain Ω . We assume that the nonlinearity f is dissipative (for instance $f(x, u) = -\beta(x)u^p$, with $\beta(x) \geq 0$) while g is explosive ($g(x, u) = u^q$) and analyze how this two mechanisms compete. We find appropriate balances between f and g that will show that the solution starting at any smooth initial condition u_0 is bounded for all time. We will show, see [1], that if these balances hold locally around certain point in the boundary, the solution is globally bounded around this point of the boundary. This result complements another one obtained in [2] in which we showed that if the balances between f and g are the opposite then blow-up occurs at that point of the boundary.

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Stability of periodic travelling wave solutions for the Korteweg-de Vries equation

by

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This paper is concerned with nonlinear stability properties of periodic travelling wave solutions of the classical Korteweg - de Vries equation,

$$u_t + uu_x + u_{xxx} = 0, \quad x, t \in \mathbb{R}.$$

It is shown the existence of a nontrivial smooth curve of periodic travelling wave solutions depending on the classical Jacobian elliptic functions. We find positive cnoidal wave solutions. Then we prove, by using the framework established in [1] by Grillakis, Shatah and Strauss, the nonlinear stability of the cnoidal wave solutions in the space $H^1_{per}([0, L])$.

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Three positive solutions for a class of fourth–order boundary value problems

by

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We study multiplicity of positive solutions for a class of fourth–order boundary value problems with non–homogeneous boundary conditions. For this, we use a fixed point theorem of cone expansion/compression type and we establish a general theorem for a type of systems of second–order ordinary differential equations involving parameters. In addition, we apply our result to the study of existence of solutions for semilinear elliptic systems in bounded annular domains.

Multiplicity of solutions for a convex-concave problem with a nonlinear boundary condition

by

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We study the existence of multiple positive solutions for a convex-concave problem with a nonlinear boundary condition involving two critical exponents and two positive parameters λ and μ of the type

$$\begin{cases} -\Delta u + u = \lambda u^{q_1} + u^{p_1} & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = \mu u^{q_2} + u^{p_2} & \text{on } \partial\Omega, \\ u > 0 & \text{in } \Omega, \end{cases} \quad (P_{\lambda\mu})$$

where $0 < q_i < 1 < p_i < \infty$ ($i = 1, 2$), $\Omega \subset \mathbb{R}^N$ ($N \geq 3$) is a smooth bounded domain and $\frac{\partial u}{\partial \nu}$ is the outer unit normal derivative.

We obtain a continuous strictly decreasing function f such that $K_1 \equiv \{(f(\mu), \mu) : \mu \in [0, \infty)\}$ divides $[0, \infty) \times [0, \infty) \setminus \{(0, 0)\}$ in two connected sets K_0 and K_2 such that problem $(P_{\lambda\mu})$ has at least two solutions for $(\lambda, \mu) \in K_2$, at least one solution for $(\lambda, \mu) \in K_1$ and no solution for $(\lambda, \mu) \in K_0$. This work is related with following papers [2], [3], [4] and [6].

By sub and super solution method [1], we obtain a minimal positive solution of $(P_{\lambda\mu})$, and we employ a version of the Ambrosetti-Rabinowitz Mountain Pass Theorem due to Ghoussoub and Preiss [5] in order to get the second positive solution.

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Boundary Stabilization of the Damped wave equation with Cauchy-Ventcel dynamic boundary conditions

by

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This work is devoted to the study of optimal and uniform decay rates of the wave equation subject to Cauchy Ventcel dynamical boundary conditions.

$$\begin{cases} u_{tt} - \Delta u = 0 & \text{in } \Omega \times]0, \infty[, \\ v_{tt} + \partial_\nu u - \Delta_\Gamma v + g(v_t) = 0 & \text{on } \Gamma_1 \times]0, \infty[, \\ u = v & \text{on } \Gamma \times]0, \infty[, \\ u = 0 & \text{on } \Gamma_0 \times]0, \infty[, \end{cases}$$

where Ω is a bounded domain of \mathbb{R}^n ($n > 2$) having a C^3 boundary $\partial\Omega = \Gamma$, such that $\Gamma = \Gamma_0 \cup \Gamma_1$, with Γ_0 and Γ_1 closed and disjoint.

We prove that the boundary dissipation $g(v_t)$ is strong enough to assure the asymptotic stability of the system. The results presented in the literature deal with localized dissipations acting in a strategic neighbourhood of the boundary (sometimes in the whole domain) in order to stabilize the system. In this paper we prove the reciprocal procedure (which remained an open problem), namely: to prove that a frictional dissipation acting on the boundary is strong enough, via transmission process ($u|_\Gamma = v$), to stabilize the whole system.

On Semi-discrete and Fully Discrete Nagamo Equations

by

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We will present complete existence and stability results for traveling waves of these equations. Some remarks on numerical schemes for conservation laws in one space dimensions will be presented.

Patterns and local minimizers for boundary reactions

by

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The existence of nonconstant stable solutions (so called patterns) for diffusion equations with some kind of nonlinear boundary reactions is an important question often related with the geometry of the domain. Basically we are going to consider

$$(1) \quad \begin{cases} u_t - \Delta u &= 0, & \text{in } \Omega \\ u_\nu &= \frac{1}{\varepsilon} f(u), & \text{on } \Gamma = \partial\Omega \end{cases}$$

for a model reaction term $f(u) = u - u^3 = u(1 - u^2)$.

In this talk we present numerical evidence of the existence of nonconstant stable equilibria for the unit square, computing families of equilibria branching off the unstable constant equilibria $u = 0$. We remark that it is the first approximation to the existence of patterns for convex domains.

The existence of equilibria can be treated as well as a variational problem and found the solutions minimizing the energy with a constraint as in the well known Ginzburg-Landau vortices phenomena. Given any $\varepsilon > 0$ we may consider solutions of (1), u_ε , which are local minima of the associated energy

$$(2) \quad E_\varepsilon(u) = \frac{1}{2} \int_\Omega |\nabla u|^2 dx + \frac{1}{\varepsilon} \int_\Gamma G(u) dl,$$

where $G' = -f$, with a constraint, that is,

$$E_\varepsilon(u_\varepsilon) = \min_{u \in X_a} E_\varepsilon(u)$$

with $X_a = \{u \in H^1(\Omega) : \|\gamma u - \chi^{p_0, q_0}\|_{L^2(\Gamma)}^2 \leq a^2\}$, and, where χ^{p_0, q_0} is the characteristic function with transition points $p_0, q_0 \in \Gamma$, alternating values -1 and $+1$.

In this way in the second part of the talk we show that we can find local minimizers as local minimizers of the renormalized energy associated to (2). As the reaction acts on the boundary these minimizers correspond to pairs of points of this boundary and the solution is obtained by harmonic extension to the interior of the domain. These patterns present a layer in the transition points, or minimizing pair, where the solution goes from -1 to $+1$ in a very small region.

Asymptotic Behavior of Parabolic Equations with Admissible Blow-up

by

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Our aim is to describe the long-time behavior of solutions of parabolic equations in the case when some of them may blow up in a finite or infinite time. This is done by providing a maximal compact invariant set attracting each initial data for which the corresponding solution does not blow up. The abstract result is applied to the Frank-Kamenetskii equation

$$\begin{aligned}
u_t &= \Delta u + \lambda e^u, & \text{in } B(0,1) \\
&+ \text{Dirichlet b.c.}, & \text{in } \partial B(0,1)
\end{aligned}$$

and to the N -dimensional Navier-Stokes system where small external force is considered.

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PDE on lattices with boundary interaction

by

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Suppose that each point on a lattice corresponds to a dynamical system generated by a PDE on a bounded domain U and there is interaction between the points on some subset V of the boundary of the domain. If it is possible to take natural limits of this interaction, then one will obtain another PDE on V . There seems to be very little literature on problems of this type. I review a paper of mine in Resenhas (1994) on transmission lines on $U=(0,1)$ with resistive coupling at the point 1 and point out some problems that need to be investigated. Not knowing how to discuss qualitative properties of the resulting PDE, special situations are given for which the dynamics can be reduced to the discussion of a partial neutral functional differential equations. We mention some generalizations of these functional differential equations and mention some other applications.

Radial Symmetry of Minimizers for a Class of Nonlocal Variational Problems

by

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We present some results about the radial symmetry of minimizers for some constrained variational problems for nonlocal functionals. For instance, consider the problem of minimizing

$$E(u, v) = \int_{R^N} \left(\frac{1}{2} |\text{grad } u|^2 + \frac{1}{2} |(-\Delta)^{1/4} u|^2 \right) dx + \int_{R^N} F(u, v) dx$$

subject to

$$Q(u, v) = \frac{1}{2} \int_{R^N} G(u, v) dx = c.$$

Under smoothness and growth assumptions on $F(u, v)$ and $G(u, v)$, and except for translation in the space variable, the minimizers are radially symmetric. There is no cooperativity assumption and we do not assume that the minimizer is positive. In that case, symmetrization cannot be used. Our approach is a combination of reflection presented in [1] with an identity involving the nonlocal term. Further examples are given.

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Vortex dynamics in domains with holes

by

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We present a new formulation of incompressible two-dimensional ideal fluid motion in terms of vortex dynamics. The difficulty is a topological one: how to write the harmonic, or potential, part of the velocity in terms of vorticity. We use this reformulation of the flow equations to describe the limiting behavior of Euler solutions in a bounded domain with multiple holes, when one of the holes becomes very small.

Patterns in Parabolic Problems with Nonlinear Boundary Conditions

by

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We obtain existence of asymptotically stable non-constant equilibrium solutions for semilinear parabolic equations with nonlinear boundary conditions on small domains connected by thin channels. We prove the convergence of eigenvalues and eigenfunctions of the Laplace operator in such domains. This information is used to show that the asymptotic dynamics of the heat equation in this domain is equivalent to the asymptotic dynamics of a system of two ordinary differential equations diffusively (weakly) coupled. The main tools employed are the invariant manifold theory and a uniform trace theorem.

Positive Solutions for a Fourth Order Equation of Kirchhoff Type

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We study the existence of positive solutions of the fourth order boundary value problem

$$u'''' - m(\int_0^1 u'(t)^2 dt) u'' = f(t, u, u'), \quad 0 < t < 1,$$
$$u(0) = u(1) = u''(0) = u''(1) = 0,$$

where m and f are positive functions. This kind of nonlocal Kirchhoff equation models the bending equilibrium of extensible beams. Our approach is based on fixed points theorems in cones of positive functions.

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Singular State-Dependent Delay Equations: Asymptotics and Stability

by

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In earlier work with Roger Nussbaum, we studied the limiting shape of solutions to singularly perturbed state-dependent delay equations, such as

$$\varepsilon \dot{x}(t) = f(x(t), x(t-r)), \quad r = r(x(t)).$$

Here we extend this work, and study the detailed asymptotics of these solutions. Such an analysis is important in obtaining stability results, as well as results for problems with multiple delays. Techniques and ideas from geometric singular perturbations are used in our analysis.

A K-theoretic proof of Boutet de Monvel's index theorem for boundary value problems

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The first goal of this talk is to explain how K-theory for C^* -algebras can be used to express index theorems for elliptic operators. Then I plan to report on a new proof of Boutet de Monvel's index theorem for boundary value problems [1] that uses K-theoretic techniques. The talk is based on joint work with Nest, Schick and Schrohe [2, 3].

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Soliton solutions for quasilinear Schrödinger equations: the critical exponential case

by

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We consider the quasilinear elliptic equation $-\Delta u + V(x)u - (\Delta(|u|^2))u = h(u)$ in \mathbb{R}^2 , where $V : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a positive potential bounded away from zero, and the nonlinearity $h : \mathbb{R} \rightarrow \mathbb{R}$ has critical exponential growth, that is, h behaves like $\exp(4\pi s^2) - 1$ as $|s| \rightarrow \infty$. Under suitable hypotheses and by assuming some asymptotic conditions on V and h , we establish an existence result for the problem above. This result is obtained by combining Ambrosetti-Rabinowitz mountain pass theorem with a version of Trudinger-Moser inequality in \mathbb{R}^2 .

On the existence of patterns in a reaction-diffusion equation with nonlinear Neumann boundary condition

by

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In this work we prove existence and determine the asymptotic profile of a family of layered stable stationary solutions (patterns, for short) to the following reaction-diffusion equation

$$(3) \quad \begin{cases} \frac{\partial u}{\partial t} = \varepsilon 2\Delta u + f(u) & \text{in } \Omega \\ \varepsilon \frac{\partial u}{\partial \nu} = \delta_\varepsilon g(u) & \text{on } \partial\Omega \\ u(0, x) = u_0(x), & x \in \bar{\Omega} \end{cases}$$

where $\Omega \subset \mathbb{R}^3$ is a C^2 simply connected bounded domain and ε a small positive parameter. It is assumed that:

- $\delta_\varepsilon \geq 1$ satisfies $\lim_{\varepsilon \rightarrow 0} \varepsilon \ln \delta_\varepsilon = \kappa$ with $0 \leq \kappa < \infty$.

- $\int_{\alpha}^{\beta} f = 0$
- $\int_{\alpha'}^{\beta'} g = 0,$

with $\alpha' \leq \alpha < \beta \leq \beta'$ and $f(\alpha) = f(\beta) = 0, g(\alpha') = g(\beta') = 0.$

Above relation holds, for example, when $\delta_{\varepsilon} = \varepsilon^{-n}, n \in \mathbb{N}$ as well as $\delta_{\varepsilon} = e^{\kappa/\varepsilon}, \kappa \geq 0.$ In particular when $\delta_{\varepsilon} = \varepsilon^{-1}$ there holds that diffusibility in Ω and $\partial\Omega$ are the same.

The equal-area conditions for f and g are actually necessary for existence of such solutions (see [2]). The nonlinear Neumann boundary condition gives rise to an involved geometric profile of the patterns, namely, the trace of the function the family of solutions approach on Ω , as $\varepsilon \rightarrow 0$, is not the function the solutions approach on $\partial\Omega$.

Main tools used are Gamma-convergence of functionals, variational techniques and results of dynamical systems in infinite dimension.

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Stable equilibrium of a diffusion equation on convex domains induced by the dynamics on the boundary.

by

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We address the question of existence of nonconstant stable stationary solution (pattern, for short) to the problem

$$(4) \quad \begin{cases} u_t = \Delta u, & (t, x) \in \mathbb{R}^+ \times \Omega \\ u(0, x) = u_0(x) & x \in \Omega \\ \partial_{\nu} u = \lambda f(u), & (t, x) \in \mathbb{R}^+ \times \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth convex domain, $\lambda \in \mathbb{R}^+$ and f a smooth assigned function.

In [1] the authors, in a computer-assisted work and using bifurcation techniques, give strong evidence that when $f(u) = u - u^3, \lambda > 2, 84083164$ and Ω the unit square (whence a convex planar domain), then (1) has a pattern.

Such solutions were known to exist for dumbbell type domains [2] and not to exist when Ω is the N -dimensional ball [3].

One way the sphere $\partial B_R(0)$ differs from any other convex hyper-surface $\partial\Omega$ is that the mean curvature of the former is constant whereas it varies in latter case. In order to explore this fact we utilize convenient coordinates to write

$$(5) \quad \Delta v = \Delta_{\mathcal{M}} v + (N - 1) H(\cdot) \partial_{\nu} v + \partial_{\nu}^2 v \quad \text{on } \mathcal{M} = \partial\Omega,$$

where $H(\cdot)$ is the mean curvature of \mathcal{M} and $\Delta_{\mathcal{M}} v = g^{\alpha\beta} u_{,\alpha\beta}$ is the Laplace-Beltrami operator with respect to the induced metric.

When the boundary condition is incorporated, we obtain the evolution equation

$$\frac{\partial u}{\partial t} = \Delta_{\mathcal{M}} u + \lambda(N - 1) H(\cdot) f(u) + \frac{\lambda 2}{2} \frac{d}{du} f^2(u) \quad \text{on } \mathcal{M}.$$

After finding a local minimizer of the functional

$$\mathcal{E}_{\mathcal{M}}(u) \stackrel{\text{def}}{=} \int_{\mathcal{M}} \left\{ \frac{\|\nabla u\|^2}{2\lambda H(\cdot)} - \left[\frac{\lambda f^2(u)}{2H(\cdot)} + (N - 1) F(u) \right] \right\} dV_g$$

where $\|\nabla u\|^2 = g^{ik}u_{,i}u_{,k} = u^{,k}u_{,k}$, $u_{,k}$ and $u^{,k}$ are respectively the covariant and contravariant components of the gradient, $F = \int_0^1 f$, problem (2) is shown to have a pattern, \bar{u} say, for λ large enough and $f(u) = u - u^3$, as long as Ω is:

- strictly convex
- symmetric with respect to a hyperplane through the origin
- $H(\cdot)$ is sufficiently small on two arbitrary disjoint sets separated by this hyperplane

Think of it as an ellipsoid whose vertical axis is much smaller than the other two.

Finally \bar{u} turns out to be the trace of the pattern for (1) we have been looking for.

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Geometry of constraints in a problem of rolling surfaces

by

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The non-holonomic mechanical problem of the rolling of a rigid ball on a surface is presented. The constraints corresponding to rolling without slipping (resp. without slipping or twisting) define a constant rank 3 (resp. 2) distribution on the 5-dimensional configuration space. Geometric properties and conditions for the controlability of these two distributions are derived.

Bifurcation of equilibria for the Chafee-Infante system on a square

by

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We study the local bifurcations from zero of equilibrium points of the dynamical system $\{\mathcal{S}_\lambda(t) : t \geq 0\}$, defined by the family of solution-operators at time- $t \geq 0$, $\mathcal{S}_\lambda(t) : \varphi \in H_0^1(Q) \mapsto \mathbf{u}(t) \in H_0^1(Q)$, where $\mathbf{u}(t)(x, y) \equiv u(t, x, y)$ is the mild solution of the semi-linear parabolic scalar equation $u_t = \Delta u + \lambda(u - du^3)$, $t > 0$, $(x, y) \in Q$, satisfying the Dirichlet boundary condition $u(t, x, y) = 0$, $t \geq 0$, $(x, y) \in \partial Q$, and the initial condition $u(0, x, y) \equiv \varphi(x, y)$, where Q denotes the square $]0, \pi[\times]0, \pi[$, ∂Q the boundary of Q , λ and d positive constants, $H_0^1(Q)$ the closure of $C_0^\infty(Q)$ in the Sobolev space $H^1(Q) = W^{1,2}(Q)$, with respect to the inner product $\langle \varphi_1, \varphi_2 \rangle_{H^1} = \langle \varphi_1, \varphi_2 \rangle_{L^2} + \langle \nabla \varphi_1, \nabla \varphi_2 \rangle_{L^2}$.

A typical result we can prove states that, when λ crosses from the left the value 50, which is an eigenvalue with multiplicity 3, of Δ on $H_0^1(Q) \cap W^{2,2}(Q)$, there appears $33 - 1 = 26$ nontrivial distinct equilibrium points near zero.

Asymptotically linear problems

by

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Our aim is to present some results on multiplicity of solutions for the semilinear problem

$$\begin{aligned} -\Delta u &= g(u) & \text{in } \Omega \\ u &= 0 & \text{on } \partial\Omega, \end{aligned}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, $g : \mathbb{R} \rightarrow \mathbb{R}$ is a function of class C^1 which is asymptotically linear at infinity. Assume that $g(0) = 0$, so $u \equiv 0$ is a solution (the trivial solution). We will give some conditions to obtain two nontrivial solutions.

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Global attractor and non homogeneous equilibria for a non local evolution equation in an unbounded domain

by

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We consider the non local evolution equation

$$\frac{\partial u}{\partial t}(x, t) = -u(x, t) + \tanh(\beta J * u(x, t) + h)$$

where $u(x, t)$ is a real function on $\mathbb{R} \times \mathbb{R}_+$, $J \in C^1(\mathbb{R})$ is a non negative even function with integral equal to 1 supported in the interval $[-1, 1]$, β, h are positive constants and $*$ denotes the convolution product.

This equation arises as a continuum limit of one-dimensional Ising spin systems with Glauber dynamics and Kac potentials; u represents then a magnetization density and β^{-1} the temperature of the system.

We show the problem is well-posed in some weighted spaces and the associated flow admits a global compact attractor. We also prove the existence of a distinguished non homogeneous equilibrium called a ‘bump solution’ or ‘critical droplet’ in the literature.

Generic simplicity for the eigenvalues of a supported plate equation

by

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Boundary perturbations have been studied by several authors through different perspectives. Among others, we mention the work Henry in [1] where the author developed a general theory on perturbation of domains and proved several results on boundary perturbations for second order elliptic operators.

Using this theory as our main tool (in particular a general form of the Transversality Theorem), we show that the eigenvalues of the problem

$$\begin{cases} (\Delta + \lambda)u(x) = 0 & x \in \Omega \\ u(x) = \Delta u(x) = 0 & x \in \partial\Omega \end{cases}$$

are simple in a residual set of \mathcal{C}^4 regular regions $\Omega \subset \mathbb{R}^n$ with $n \geq 2$.

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Morse-Smale Attractors for Semilinear Parabolic Equations

by

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We survey the existing results on the characterization of global attractors for semiflows generated by scalar semilinear parabolic equations under periodic boundary conditions. In particular we describe a characterization of all the Morse-Smale attractors obtained using a permutation characterization developed for Neumann boundary conditions.

On The Well-Posedness for the generalized Ostrovsky, Stepanyams and Tsimring equation

by

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In this talk we consider the initial value problem (IVP) associated to equation

$$u_t + u_{xxx} - \eta(\mathcal{H}u_x + \mathcal{H}u_{xxx}) + u^k u_x = 0, \quad x \in \mathbb{R}, t \geq 0,$$

where $\eta > 0$, and \mathcal{H} denotes the usual Hilbert transform. We will describe the local results obtained for the IVP in Sobolev spaces $H^s(\mathbb{R})$ for $s \geq 0$ and $k = 1, 2, 3$ and the global ones in $L^2(\mathbb{R})$.

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Existence and multiplicity of solutions for singular elliptic problems

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In this talk we present results on the existence and multiplicity of positive solutions for singular problems. The method combines a perturbation argument and critical point theory.

Some improvements of Hölder results

by

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We investigate some limit cases of Hölder inequality for sums and integrals and establish some relations between divergence rates for corresponding sums and integrals.

Laplace's equation in the half-space with nonlinear boundary conditions

by

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The talk will deal with stable nonconstant solutions of Laplace equation in a half-space with nonlinear boundary conditions, or the equivalent problems for the energy functional.

We will describe the recent results of the paper [1], concerning existence, uniqueness and stability properties for solutions to this problem that are monotonic in some direction parallel to the boundary.

The knowledge of these monotonic solutions, also called layer solutions, has also been used recently to obtain some more results on the existence of stable nonconstant solutions for this problem in bounded domains. These results, that are due to X. Cabré and N. Cònsul will also be commented, specially in relation with previously known results.

In all of these works on bounded domains, one of the main goals has been to relate the geometry of the domain (a domain with holes, a ball, a dumb-bell shaped domain, a square, or others...) with the property of the existence or not of these stable nonconstant solutions.

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Invariant varieties of discontinuous vector fields

by

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We study the geometric qualitative behaviour of a class of discontinuous vector fields in four dimensions around typical singularities. We are mainly interested in giving the conditions under which there exist one-parameter families of periodic orbits (a result that can be seen as one analogous to the Lyapunov centre theorem). The focus is on certain discontinuous systems having some symmetric properties. We also present an algorithm which detects and computes periodic orbits.

Almost periodically forced circle flows

by

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Almost automorphy is a notion first introduced by S. Bochner in 1955 to generalize the almost periodic one. It is proven to be a fundamental notion in characterizing multi-frequency phenomena and their generating dynamical complexity in almost periodically forced monotone systems. The lecture will present some recent results on almost automorphic dynamics arising in almost periodically forced circle flows that are generated from almost periodically forced nonlinear oscillators and Hamiltonian systems.

LATE ARRIVALS

Oceanic dynamics

by

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This lecture will be an introduction to the mathematical theory of oceanic fluid flows, with emphasis on the longtime dynamics of the models. Some applications will be included.

Large time behaviour of Neutral delay systems

by

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We are interested in *functional differential equations* (FDE) of the form

$$(6) \quad \begin{aligned} \frac{d}{dt} Mx_t &= Lx_t, \\ x_0 &= \varphi \in \mathcal{C}. \end{aligned}$$

where M and L are linear continuous operators from the space state $\mathcal{C} \stackrel{\text{def}}{=} \mathcal{C}([-1, 0], \mathbb{C}^n)$ to \mathbb{C}^n , and $x_t \in \mathcal{C}$ is defined as

$$x_t(\theta) = x(t + \theta), \quad \theta \in [0, 1], \quad t \geq 0.$$

We aim to show a decomposition of the space state $\mathcal{C} \stackrel{\text{def}}{=} \mathcal{C}([-1, 0], \mathbb{C}^n)$ as direct sums of $\mathcal{M}_\lambda \oplus \mathcal{Q}_\lambda$, where \mathcal{M}_λ is a finite dimensional subspace of \mathcal{C} , and estimates that reduce the large time behaviour of solutions of the initial value problem (6) to an ordinary differential equation in \mathbb{C}^n .

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